Capital Estimation: Theory and US-Japan Comparative Study

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Abstract: Capital stock estimation is a thorny task to deal with. To circumvent the problem, we derive a function which does not contain capital stock, yet whose regression indirectly estimates the production function. Consequently, the estimated production function can generate the capital stocks and depreciation rates. The approach applied to US data resulted in estimates of both production function and capital stock very close to those differently estimated in existing literature. Japan data are then applied as a comparative study.

Keywords: Capital stock, depreciation rate, indirect estimation, production function, data generation.

1. INTRODUCTION

Economists worldwide are interested in estimating capital stocks for a variety of reasons and purposes. Nevertheless, capital stocks estimates are very sensitive to the underlying assumptions and the definition of capital stock. As a result, the estimated capital/output ratios vary greatly from one paper to the next. Hence, it can be said that capital stock is one of the most elusive and agonizing economic variables to approximate.

In this paper, we introduce a fresh and simple econometric approach to estimation of capital stock almost free of assumptions and operational definitions normally required in conventional approaches. The new approach not only can generate a bench mark capital stock series but also save some sticking points associated with conventional approaches to estimating the capital stock. Data for the US and Japan are then analyzed for comparative studies and identify each country's specific characteristics. Preliminary tests reveal not only a statistically significant heteroskedasticity but also a statistically significant autocorrelation of first order. Hence, the standard errors were adjusted based on Newey and West [1]. We then use the estimation results to generate two datasets of capital stocks for the US and Japan over time and compare the two countries to each other.

Section 2 of this paper reviews the existing literature. Section 3 introduces the model. Section 4 carries out data analysis and generation, and Section 5 concludes.

2. EXISTING LITERATURE

The earliest attempt to estimate capital stock is shown in Ward [2], who introduces the concept of average life, which is the average period of time during which the capital remains in stock. Based on this average life, the author estimates the average rate of depreciation of capital. Once the average rate of capital depreciation is estimated, the subsequent capital stock series is then calculated by subtracting depreciation values from the accumulated values of investment. Also trying to determine the rate of capital depreciation but using a different approach, Hulten and Wykoff [3] apply the Box–Cox transformation and analyze data on used asset prices. The Box–Cox model is employed to statistically distinguish different patterns of capital depreciation. The results show that the appropriate depreciation pattern is approximately geometric. The subsequent estimation of capital stocks are then calculated using the same method used in Ward [2]. Another method of estimating the rate of capita depreciation is presented in Hulten, C.R. [4], as well as Nadiri and Prucha [5] and is based on a weighting system of present and past investment values. Once the average rate of capital depreciation is estimated, the values of capital stocks are calculated as a weighted sum of current and lagged investment values. On the contrary, Prucha [6] uses dummy variables to estimate this rate of depreciation. The subsequent series of capital stocks then are calculated using the same method used in Hulten, C.R. [4].

Dadkiah and Zahedi [7] use the correlation between capital stock growth and output growth to estimate the growth rate of capital stock based on the production function equation. Solving for capital growth rate as a dependent variable of labor and output growth from this equation, they obtain a time series dataset for the capital stocks of 69 countries and use the capital stocks series to forecast the future output growth for

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these countries. Instead of estimating capital stock, Summers and Heston [8] make an important contribution to the literature by expand the original time series data on consumption expenditures, investment, and government purchases for the period 1970-1985 in the Pen World Table to a much longer time period that covers from 1950 to 1988. This dataset is later used by several authors to estimate capital stocks or the total factor productivity for various countries.

Gábor Pula [9] calculates the average capital output ratio for US during 1980-1990 period based on the data in Summers and Heston [8] and finds that it is approximately 0.9. This paper also shows that capital/output ratios worldwide varies greatly, ranging from 0.8 to 3.2. Although Gomme and Rupert [10] mainly analyze labor's share of income, they roughly provide the capital's share of US income during 1950 to 2000 period by subtracting its labor's share of income from the total income. They find that the average capital's share of US income is 0.265 including taxes for 1970-2004. Not directly related to the capital's share of income are three papers that use the results in the aforementioned literature to estimate the annual growth rate of total factor productivity: Nehru and Dhareswar [11] find that this growth rate is roughly 0.011 for 1960-1987; Fernald and Ramnath [12] show that it is around 0.012 for 1948-2000, and 0.0107 OECD manuals [13] confirms that it is close to 0.011 for 1960-1995.

Im and Vu [14] are the first authors to substitute the interest rate into the production function equation and solve for capital's share of income as a function of the interest rate, a time trend, and output. Nonetheless, they then use the growth accounting equation to find out the contribution of capital to the output growth instead of deriving a formula for generating a dataset for capital stocks. Specifically, they substitute the coefficient of capital share into the accounting growth equation to examine the effect of capital stock growth on output growth. Additionally, that paper focuses on a specific case study of Vietnam's growth accounting. The authors find that capital growth accounts for 33.59% of value-added growth per person in Vietnam during 1990-2002 but only 19.94% of this growth during 2000-2010.

In brief, none of the existing papers use the method introduced in this paper to obtain a dataset for capital stocks. In the following sections, we discuss in details our model and data analyses for the US and Japan.

3. THE MODEL

To start with, we employ the Cobb-Douglas production function with a constant return to scale for its long established status in empirical study in macroeconomics:

$$Y_t = A e^{\tau t} K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where $\theta \equiv (A, \tau, \alpha)'$ is an unknown parameter vector whose estimation requires data on Y_t, K_t, and L_t. In this context, suppose we can estimate θ without using (1), then K_t can be generated from Equation (1) as long as we have observations on Y_t and L_t only. To show it, we first rewrite Equation (1), reflecting the assumed constant returns to scale (henceforth with subscript t suppressed until necessary to avoid possible confusion):

$$y = A e^{\tau t} k^{\alpha} \tag{2}$$

where $y = Y / L$ and $k = K / L$.

Then

$$\Delta y = \frac{\partial y}{\partial k} \Delta k = A \alpha e^{\tau t} k^{\alpha-1} \Delta k \ .$$

where Δk denotes per capita capital formation or investment and Δy the corresponding change in output or income.

Since investment at the macro level in any time period is made ultimately at the expense of the *same amount* of consumption in the same period, Δk not only represents investment but simultaneously represents the opportunity cost in terms of consumption foregone. Therefore, the net return to investment Δk, denoted by π below, must equals to the difference between Δy (output due to investment Δk) and Δk (cost of Δk in terms of the consumption foregone) so that

$$\pi = \Delta y - \Delta k = A e^{\tau t} \alpha k^{\alpha-1} \Delta k - \Delta k = \left(A e^{\tau t} \alpha k^{\alpha-1} - 1 \right) \Delta k \ .$$

Hence, the rate of return of investment Δk is expressed as

$$\rho \equiv A e^{\tau t} \alpha k^{\alpha-1} - 1 \tag{3}$$

If investment in each time period is to maximize the return, investment must expand up to the point where the rate of return equals the real interest rate (r), i.e., $\rho = r$, so that ρ in Equation (3) can be replaced by

$$r = Ae^{\tau t}\alpha k^{\alpha-1} - 1. \tag{4}$$

Solve for *k*:

$$k = \left(\frac{1+r}{Ae^{\tau t}\alpha}\right)^{\frac{1}{\alpha-1}} = \left(A^{-1}e^{-\tau t}\alpha^{-1}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{1}{\alpha-1}}$$
$$= A^{\frac{1}{1-\alpha}}e^{\frac{\tau}{1-\alpha}t}\alpha^{\frac{1}{1-\alpha}}(1+r)^{\frac{1}{\alpha-1}} \tag{5}$$

Substitute k into Equation (2) and solve for y:[1]

$$y = A^{\frac{1}{1-\alpha}}e^{\frac{\tau}{1-\alpha}t}\alpha^{\frac{\alpha}{1-\alpha}}(1+r)^{\frac{\alpha}{\alpha-1}} \tag{6}$$

Take the logarithm of Equation (6):

$$\ln y = \beta_o + \beta_1 t + \beta_2 \ln(1+r) \tag{7}$$

where

$$\beta_o \equiv \frac{1}{1-\alpha}\ln A; \quad \beta_1 \equiv \tau/(1-\alpha); \quad \beta_2 \equiv \alpha/(\alpha-1) \tag{8}$$

The stochastic version of Equation (7), with the usual assumption that the residual is independent of the regressors t and $(1+r)$, can be expressed in terms of matrices as

$$Z = \beta X + \varepsilon; \quad \varepsilon \sim N(0,\sigma^2\Omega); \quad E(X'\varepsilon) = 0 \tag{9}$$

in which

$$Z \equiv \begin{pmatrix} \ln y_1 \\ \ln y_2 \\ \vdots \\ \ln y_T \end{pmatrix}; \quad X \equiv \begin{bmatrix} 1 & 1 & \ln(1+r_1) \\ 1 & 2 & \ln(1+r_2) \\ \vdots & \vdots & \vdots \\ 1 & T & \ln(1+r_T) \end{bmatrix};$$

$$\beta \equiv \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}; \quad \varepsilon \equiv \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_T \end{pmatrix}$$

Let $\hat{\beta} = (\hat{\beta}_o, \hat{\beta}_1, \hat{\beta}_3)'$ denote the OLS estimator of β in Equation (9). Then, since we know $E(\hat{\beta}) = \beta$ under the assumption of $E(X'e) = 0$, we can state

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[1]If we ignore the opportunity cost of investment, the rate of return to investment becomes $r = \Delta y / \Delta k = Ae^{\tau t}\alpha k^{\alpha-1}$, consequently $y = A^{\frac{1}{1-\alpha}}e^{\frac{\tau}{1-\alpha}t}\alpha^{\frac{\alpha}{1-\alpha}}r^{\frac{\alpha}{\alpha-1}}$ which did poorly in regression vis-à-vis (6).

$$E(\hat{\beta}_o) = \frac{1}{1-\alpha}\ln A; \, E(\hat{\beta}_1) = \beta_1 = \tau/(1-\alpha); \, E(\hat{\beta}_2) = \beta_2 = \alpha/(\alpha-1) \tag{10}$$

Let $\theta \equiv (A,\alpha,\tau)'$ and $\tilde{\theta} \equiv (\tilde{A},\tilde{\alpha},\tilde{\tau})'$ such that

$$\hat{\beta}_o = \frac{1}{1-\tilde{\alpha}}\ln\tilde{A}; \quad \hat{\beta}_1 = \tilde{\tau}/(1-\tilde{\alpha}); \quad \hat{\beta}_2 = \tilde{\alpha}/(\tilde{\alpha}-1) \tag{11}$$

Solve Equation (11) for each element of $\tilde{\theta} = (\tilde{A},\tilde{\alpha},\tilde{\tau})'$ in terms of elements of $\hat{\beta}$,

$$\tilde{A}(\hat{\beta}) = e^{\frac{\hat{\beta}_o}{1-\hat{\beta}_2}}; \quad \tilde{\alpha}(\hat{\beta}) = \hat{\beta}_2/(\hat{\beta}_2-1); \quad \tilde{\tau}(\hat{\beta}) = \hat{\beta}_1/(1-\hat{\beta}_2) \tag{12}$$

Since $E(\hat{\beta}) = \beta$ implies $p\lim\hat{\beta} = \beta$,

$$p\lim\tilde{A}(\hat{\beta}) = p\lim e^{\frac{\hat{\beta}_o}{1-\hat{\beta}_2}} = e^{\frac{p\lim\hat{\beta}_o}{1-p\lim\hat{\beta}_2}} = e^{\frac{\beta_o}{1-\beta_2}} = A;$$

$$p\lim\tilde{\alpha}(\hat{\beta}) = p\lim\hat{\beta}_2/(p\lim\hat{\beta}_2-1) = \beta_2/(\beta_2-1) = \alpha$$

$$p\lim\tilde{\tau}(\hat{\beta}) = p\lim\hat{\beta}_1/(1-p\lim\hat{\beta}_2) = \beta_1/(1-\beta_2) = \tau \tag{13}$$

where the first equalities are in light of the law of large numbers.

System (13) shows that each element of $\tilde{\theta}(\hat{\beta}) = (\tilde{\alpha}(\hat{\beta}), \tilde{\tau}(\hat{\beta}), \tilde{A}(\hat{\beta}))'$ are a consistent estimator of its counterpart of $(A,\alpha,\tau)'$, the parameter vector underlying the production function in Equation (1) and provides a theoretical foundation for use of $\tilde{\theta}(\hat{\beta})$ as an estimator of θ. Hence, using $\tilde{\theta}(\hat{\beta})$ for θ in Equation (1), we derive a consistent estimator of K_t given L_t and Y_t:

$$\hat{K}_t = \tilde{A}^{-\frac{1}{\tilde{\alpha}}}e^{-\frac{\tilde{\tau}}{\tilde{\alpha}}t}L_t^{\frac{\tilde{\alpha}-1}{\tilde{\alpha}}}Y_t^{\frac{1}{\tilde{\alpha}}} \tag{14}$$

Also, estimating depreciation rates has been an important subject matter of economic research. In our approach, once capital stock are generated by Equation (14), we can generate the time-varying depreciation rate over time based on

$$\hat{d}_t = \frac{(\hat{K} + I_{t+1}) - \hat{K}_{t+1}}{\hat{K}_t} = 1 - \frac{\hat{K}_{t+1} - I_{t+1}}{\hat{K}_t} \tag{15}$$

4. EXPERIMENT WITH US AND JAPAN DATA

We estimated Equation (9) for US and Japan. The annual time series data for real GDP from 1977 to 2011 are from the US Department of Agriculture (USDA)

Table 1: OLS Estimation for the US with the Newey-West Standard Error Adjustment

Variable	Coefficients	Standard Error†	p-value
Trend	0 .0152*	0.0004	0.000
Log of Real Interest	-0.4668*	0.1654	0.008
Intercept	10.936*	0.8784	0.000
Sample Size	35		
Prob. > F	0.000		
Adjusted R-Square	0.9896		
White test: p-value	0.0297		
Autocorrelation coefficient	0.7281 (p-value = 0.0000)		

Notes: *denotes statistically significant at 1 % level.
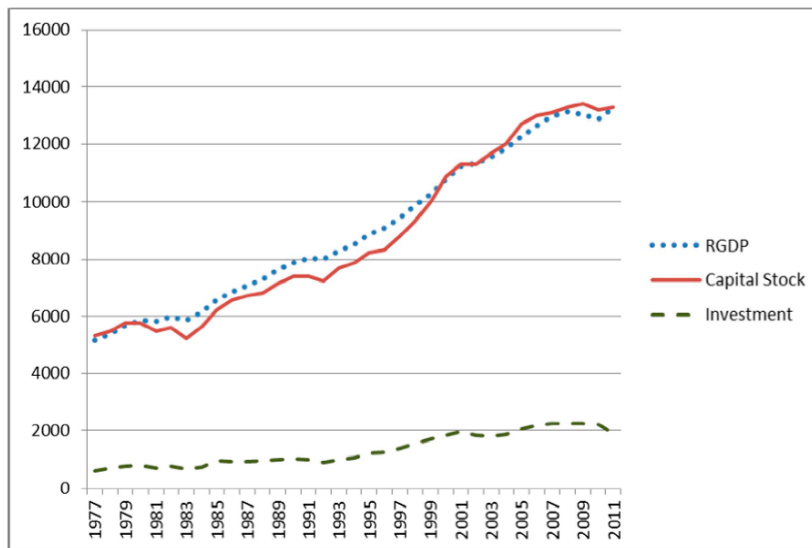
Figure 1: Estimated Capital Stock for the US in Comparison with Output and Investment.

Data Base. Data for the interest rate and employment are from the International Monetary Fund (IMF) International Financial Statistics, updated by economagic.com. For the interest rates, we used the rate of return on one-year government bonds. For the investments, we use the domestic investment from each country's national accounts. To control for the business cycle effects, we calculate double moving averages of the estimated series before generating the capital stock datasets.

We first examine the US data. The OLS estimation of β resulted in $\hat{\beta} = (10.93602, 0.0152, -0.4668)'$ each element of which statistically significant with 1% significance level. We were not able to reject the null hypothesis of no endogeneity for $\ln(1+r_t)$, the only potentially endogenous regressor in Equation (9),[2]

which statistically confirms that $\hat{\beta}$ is an unbiased estimator of β. From the OLS residuals, however, we find not only a statistically significant heteroskedasticity but also a significantly significant autocorrelation of first order as shown in Table **1**. While these problems do not bias $\ln(1+r_t)$ in the context of $E(X'\varepsilon) = 0$ as confirmed, they do bias its standard errors and their related test statistics. Hence, the standard errors were adjusted for both heteroskedasticity and autocorrelation based on Newey and West (1987) as reported in Table **1**, which also shows that estimated coefficients are significant at 1% level.

Feeding elements in $\hat{\beta}$ into (12), we obtain. $\tilde{\theta}(\hat{\beta}) = (\tilde{A}, \tilde{\tau}, \tilde{\alpha})' = (1728.63, 0.010, 0.3183)'$.

$\tilde{\alpha} = 0.31$, which estimates the capital share of income, is very close to the average capital share of US income during 1950 to 2000 period in Gomme and Rupert [11]. $\tilde{\tau} = 0.010$, estimate of annual growth rate of the total factor productivity, is very close to 0.012 for

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[2]For endogeneity test, we regress $\ln(1+r_t)$ on trend variable t to obtain the residuals ($\hat{u}_t$). We then regress Equation (10) with $\hat{u}_t$ added. The estimated coefficient for $\hat{u}_t$ is insignificant with 0.007 (p-value = 0.314).

Table 2. OLS Estimation for Japan with the Newey-West Standard Error Adjustment

Variable	Coefficients	Standard Error†	p-value
Trend	0.0155*	0.017	0.000
Log of Real Interest	-0.6072**	0.2744	0.023
Intercept	13.038*	2.384	0.000
Sample Size	35		
Prob. > F	0.000		
Adjusted R-Square	0.8473		
White test: p-value	0.0126		
Autocorrelation coefficient	0.4742 (p-value = 0.006)		

Notes: * and **denote statistically significant at 1 % level and 5 % level, respectively.

1948-2000 in Fernald and Ramnath [12] and 0.0107 for 1960-1990 in Nehru and Dhareswar [8]. $\hat{K}_t$ (estimated capital stock) is plotted in Figure **1** in reference to Y_t (actual output) and I_t (actual investment) for the US.

$\hat{K}_t$ and Y_t move quite close to each other though capital stock is somewhat less than the output for most part of the sample period visibly except 2003 to 2007 sub sample period. Pula [10] finds that the average capital output ratio for US during 1980-1990 period based on the data in Summers and Heston [6] is approximately 0.9 which is quite close to 0.94 based on the our estimated capital stocks for the same period.

We then estimated Equation (9) for Japan annual data from 1977 to 2011 as a comparative study. The results are reported in Table **2** and reveal the similarity to those in Table **1**.

However, from this table the calculated income share of capital for Japan, which is 0.38, is much

higher than that of the US (= 0.31). This fits the reality where Japan's saving and investment rates are much higher than those in the US. Figure **2** pots the estimated capital stock for Japan.

Although capital and output in Japan also move closely for the entire sample period, it is interesting to see that Japan's stock of capita was much higher than its output during 1980s and early 1990s, implying an overinvestment behavior that lead to the consequence of the so called "bubble economy" in Japan during this period and the crisis in the mid and late 1990s. This overinvestment of Japan during 1980s was more severe to that of the US during 2005-2008, when the US's stock of capital was only slightly higher than its output. As a result, the correction in Japan also appeared to be more pronounced than that in the US: the former's capital stock remained lower than its output throughout 1997-2007 whereas the latter's capital stock was only falling for two years.
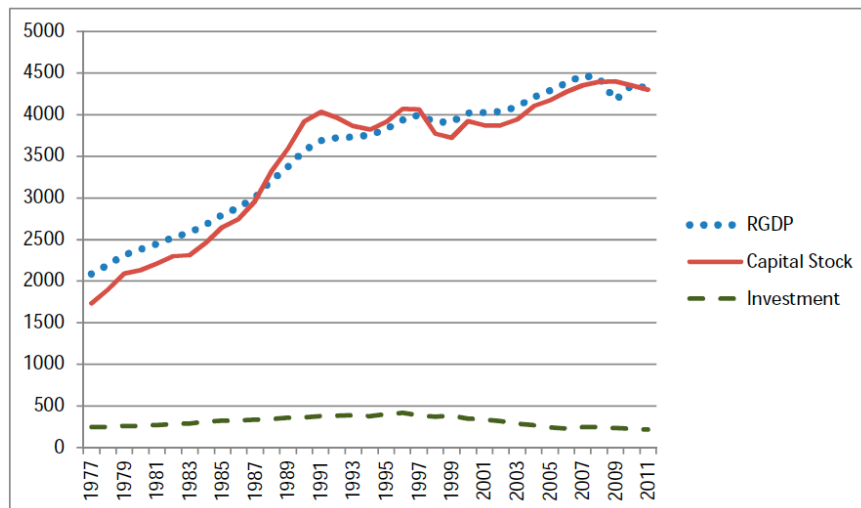
Figure 2: Estimated Capital Stock for Japan in Comparison with Output and Investment.

5. CONCLUSION

This paper shows that the parameters underlying production function can be indirectly estimated without capital stock data. This allows us to generate the capital stock over time and even the time-varying depreciation rate if need arises. The new approach not only can generate a bench mark capital stock series but also save some sticking points associated with conventional approaches to estimating the capital stock. Data analyses for the US and Japan show similar results and identify each country's specific characteristics, implying the robustness of the theoretical model. It is also interesting to apply the capital data generated using this model into any empirical study on the effect of capital on real GDP or productivity but is not the focus of this paper.

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