Determination of Weibull Parameter by Four Numerical Methods and Prediction of Wind Speed in Jiwani (Balochistan)

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Abstract: In this paper we determine the Weibull parameters (k and c) using four methods. Method of Moments, Empirical Method, Energy Pattern Method and Maximum likelihood Method have been employed to determine and compare the values of k and c. The daily wind speed data (obtained from Karachi Meteorological office) of Jiwani (Balochistan, Pakistan) town over a period of 10 years (1998-2007) is used to estimate the Weibull parameters. The mean wind speed was predicted using k and c by four methods. A significant agreement is found in measured and predicted mean wind speed.

Keywords: Wind energy, Weibull distribution, Weibull Parameters.

INTRODUCTION

Energy is essential for every living organism. The quality of human life as well as economic growth and activities are linked to energy. The continuous and inexpensive supply of energy is a need of every individual. The HDI of developed nations is above 0.8 and a high per capita energy consumption whereas developing nations have HDI below 0.5 and a low per capita energy consumption. For a country the economic development is indicated by per capita energy consumption [1].

At present the world is confronted with shortage of energy due to growing demand, it is expected that in next 20 years the demand could be double. For the developing countries including Pakistan the situation is much worst (a great challenge for them). The development of Pakistan depends on availability of useable, continuous and inexpensive supply of energy. Presently, the energy crises lead to explore renewable resources.

In order to generate low-cost energy wind is considered to be the most abundant, inexhaustible and clean source of energy. Nowadays a rapid growth is found in wind power utilization [2-4]. The worldwide, potential of wind energy has reached to 237016 MW, 40053 MW of which were put in 2011. The growth rate of Wind energy has reached to 20.3%. Ninety six countries have been generating electricity by wind power. In 2011, 500 TerraWatt hour per annum were added with the help of wind turbines, which is 3% of the electricity consumption, worldwide. A 50 billion Euro of turnover was observed in 2011 in the wind sector, the contribution of different parts is as under:

Asia comes first; their contribution due to new installations is 57.3%, Europe come second with a contribution of 21.9%, third is North America, whose contribution is almost close to that of Europe (20.5%). Latin America and Australia/Oceania have contribution of 2.9% and 0.9%, respectively. The global capacity of wind power utilization may reach to 500,000 Megawatts by the year 2015 and more than 1,000,000 Megawatts by the year 2020 [5].

Weibull Distribution

The most common density/distribution functions are the Rayleigh and Weibull density/distribution functions which are used to illustrate the wind speed data [6]. The Weibull distribution has been playing an important role to model and predict wind power and distributions. Weibull distribution is very flexible and easy to apply. Fortunately, the wind data behaves almost in similar fashion as Weibull distribution does. The simplest form of Weibull distribution has two parameters: the parameter **'k'** (known as scale parameter (dimensionless) and 'c' the scale parameter, c has dimension of velocity [7].

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Like other density functions Weibull distribution also has a probability distribution / density function (PDF) and cumulative distribution / density function (CDF).

Probability Density Function (PDF)

Weibull Probability density function has been used by a number of researchers for modeling and predicting wind energy [8-14]. The wind speed is used as a continuous random variable of the distribution. The mathematical form of PDF is:

$$f(v)=\frac{k}{c}\left(\frac{v}{c}\right)^{k-1}e^{-\left(\frac{v}{c}\right)^{k}} \tag{1}$$

Where k is a dimensionless quantity and is known as shape factor or parameter which indicates wind stability and related to the variance of the wind speed and **c** is a scale factor or parameter and is associated with mean wind speed.

Cumulative Distribution Function (CDF)

$$f(v)=1-e^{-\left(\frac{v}{c}\right)^{k}} \tag{2}$$

Effect of Weibull Shape Parameter k and Scale Parameter c

The shape parameter 'k' helps in finding how frequently wind speeds are close to some measured speed. The median value of distribution means indicates equal distribution on both the sides, i.e. 50 % wind speeds are more than the median and 50% are less than it. The value of k represents variation in mean wind speed in a given sample; higher the value of k more the stability in wind speed. The scale parameter (c) is an indicator of wind potential of a place. The large value of 'c' is an indication of high wind power [15].

Determination of Parameters

The parameters of Weibull distribution's parameter (k and c) can be found by a number of ways. Some of them are mentioned here.

(i) Maximum Likelihood Method (MLM)

MLM was suggested by Steven *et al*. [16]. It adopts an iterative procedure for determination of parameters, k and c. These parameters are found by the eqs. (3) and (4).

$$k=\left[\frac{\sum_i^n v_i^k \ln(v_i)}{\sum_i^n v_i^k}-\frac{\sum_i^n \ln(v_i)}{n}\right]^{-1} \tag{3}$$

$$c=\frac{\sum_i^n v_i^k}{n} \tag{4}$$

A special care must be taken in estimating Weibull parameters through MLM for wind data that includes zero wind speed. The reason is clear from the shape parameter equation that involves logarithm; Logarithm of zero makes the calculation indeterminate. An initial value of k = 2 is suitable to start the iteration.

(ii) Methods of Moments (MoM)

The method of moments is considered as an alternative to maximum likelihood method. The first two moments of the Weibull density function are utilized to calculate the parameters 'k' and 'c'. The calculations are based on standard deviation, average wind velocity and gamma function for parameter (1+1/k). This method is suggested by Jestus *et al.* [17]. The two moments of the distribution are given in equations (5) and (6) which help in calculating shape and scale parameters.

$$\overline{v}=c\Gamma(1+1/k) \tag{5}$$

$$\sigma=c[\Gamma(1+1/k)-\Gamma^2(1+1/k)]^{1/2} \tag{6}$$

(iii) Empirical Method (EMP)

Empirical method, like method of moments, uses standard deviation and mean wind speed for determining Weibull parameters.

The shape parameter **k** is evaluated by eq. (7).

$$k=\left(\frac{\sigma}{v}\right)^{-1.086} \tag{7}$$

And the scale parameter is evaluated by eq. (8).

$$\overline{v}=c\Gamma(1+1/k) \tag{8}$$

(iv) Energy Pattern Factor Method (EPM)

EPM also known as power density method, is simple and easy to implement. It uses average of wind speed cubes $(\overline{v^3})$ and cube of average wind speed $(\overline{v^3})\cdot\frac{\overline{v^3}}{(\overline{v})^3}$ is known as energy pattern factor (E_{pf}). The scale factor is determined directly from energy pattern factor. The equations for finding scale parameters are the same as those used for method of moments and empirical method. These equations are given below:

$$E_{pf}=\frac{\overline{v^3}}{(\overline{v})^3} \tag{9}$$

$$k = 1 + \frac{3.69}{(E_{pf})^2} \quad (10)$$

$$\bar{v} = c\Gamma(1 + 1/k) \quad (11)$$

RESULTS AND DISCUSSION

The exceeding demand of alternate energy has opened a field of research that deals with renewable energy. Theoretical work has been carried out to discover a reliable method for the estimation of wind speed. Weibull distribution has been used extensively to predict wind energy potential. The reliability of the prediction depends on how precisely the parameters k and c are determined [11]. Various methods have been employed to determine the Weibull parameters [7, 10-11]. Some of them are "the graphical method, the moment method, the maximum likelihood method, the energy pattern factor method, the empirical method, the least square method, the modified maximum likelihood method and the equivalent energy method" [18]. In this study we compared Weibull parameters which are computed by four different methods given above:

Jiwani (Blaochistan, Pakistan)

Jiwani is located on the Makran coast in the Gwadar District along the Gwadar bay (see Figure **1**). It is a commercial port situated near the Iran-Pakistan border [19]. It is basically a town of Balochistan province of Pakistan. Jiwani, like some other costal-areas of Paksitan, has a high wind potential for the generation of electrical energy.

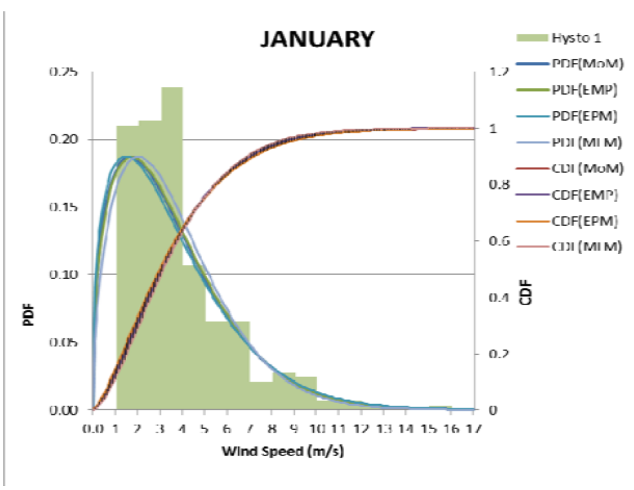
Figure 1: Map of Jiwani Town.

Table 1: The Estimated and Monthly Mean Wind Speed Averaged Over 10 Years (1998-2007).

Months	Mean wind speed / (m/s)					Absolute Error in mean speed			
	Observed	MoM	EMP	EPM	MLM	MoM	EMP	EPM	MLM
Jan	3.58	3.298	3.314	3.252	3.537	0.286	0.270	0.331	0.046
Feb	4.12	4.035	4.044	4.021	4.079	0.089	0.081	0.104	0.046
Mar	4.60	4.562	4.569	4.552	4.595	0.041	0.035	0.051	0.009
Apr	4.70	4.696	4.697	4.692	4.711	0.009	0.008	0.013	0.006
May	4.64	4.650	4.650	4.648	4.656	0.013	0.014	0.012	0.019
Jun	4.99	4.981	4.981	4.978	4.989	0.006	0.005	0.008	0.002
Jul	5.13	5.140	5.141	5.137	5.156	0.007	0.008	0.004	0.023
Aug	5.07	4.926	4.927	4.925	4.942	0.145	0.144	0.146	0.129
Sep	4.05	3.888	3.889	3.885	3.893	0.159	0.158	0.162	0.154
Oct	3.43	3.425	3.426	3.424	3.438	0.009	0.008	0.010	0.004
Nov	3.17	3.158	3.160	3.158	3.167	0.009	0.008	0.010	0.001
Dec	2.81	2.777	2.782	2.768	2.815	0.036	0.032	0.046	0.001
Mean	4.19	4.13	4.13	4.12	4.16	0.064	0.061	0.072	0.027

Table 2: Comparison of Scale and Shape Parameters

1998-2007	k				c (m/s)			
	MoM	**EMP**	**EPM**	**MLM**	**MoM**	**EMP**	**EPM**	**MLM**
Jan	1.4205	1.4421	1.3631	1.5392	3.9396	3.9483	3.9137	4.0108
Feb	1.7352	1.7604	1.6954	1.8005	4.6283	4.6322	4.6213	4.6679
Mar	1.7798	1.8049	1.7437	1.8390	5.1895	5.1932	5.1836	5.2206
Apr	2.1639	2.1856	2.1045	2.1752	5.3119	5.3119	5.3114	5.3285
May	2.4343	2.4510	2.3030	2.3862	5.2450	5.2442	5.2497	5.2540
Jun	2.3831	2.4009	2.3257	2.3717	5.6251	5.6244	5.6272	5.6356
Jul	2.3929	2.4105	2.3357	2.3950	5.8083	5.8075	5.8105	5.8258
Aug	2.4173	2.4344	2.3745	2.4259	5.5602	5.5594	5.5620	5.5776
Sep	2.2275	2.2481	2.1165	2.2197	4.3936	4.3933	4.3937	4.3992
Oct	2.2240	2.2447	2.1946	2.2456	3.8767	3.8765	3.8769	3.8904
Nov	2.0288	2.0522	2.0146	2.0572	3.5745	3.5751	3.5741	3.5835
Dec	1.5572	1.5812	1.5108	1.6521	3.1291	3.1340	3.1187	3.1732



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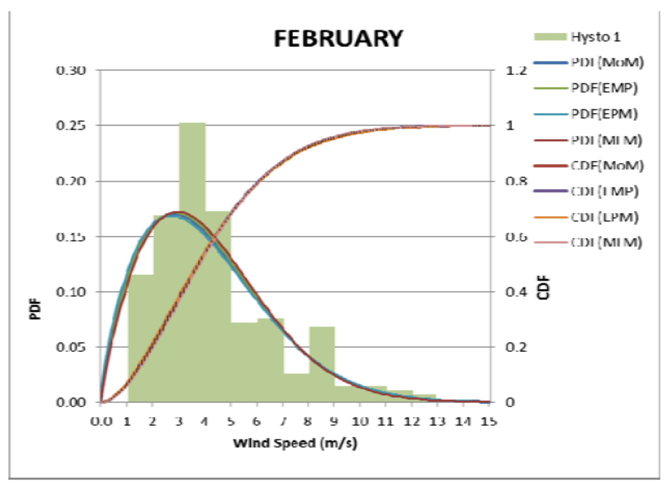

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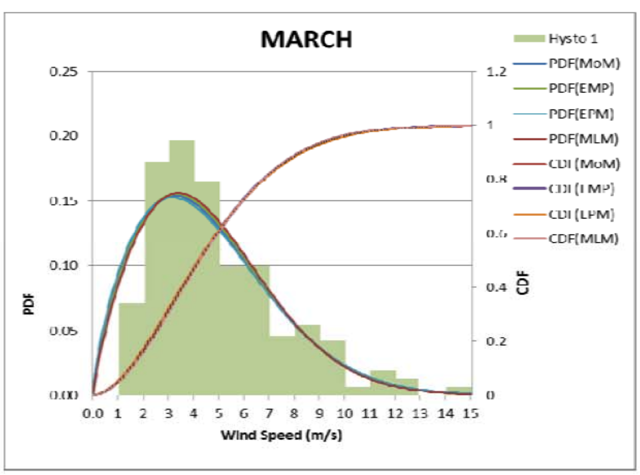

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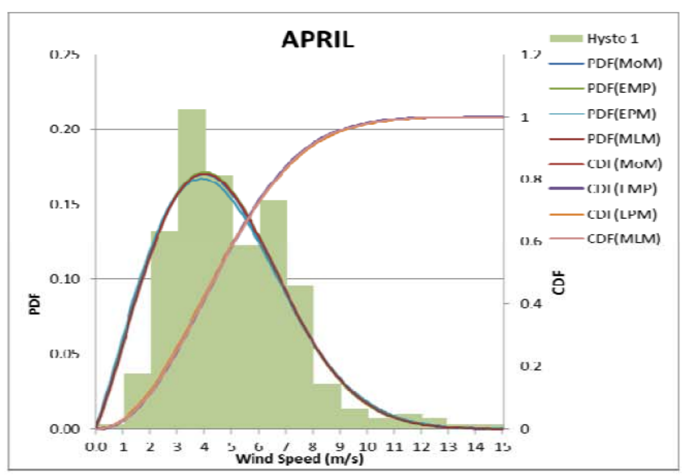

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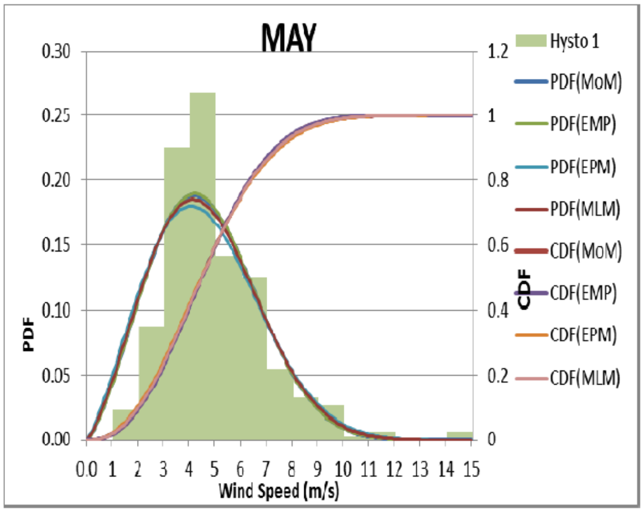

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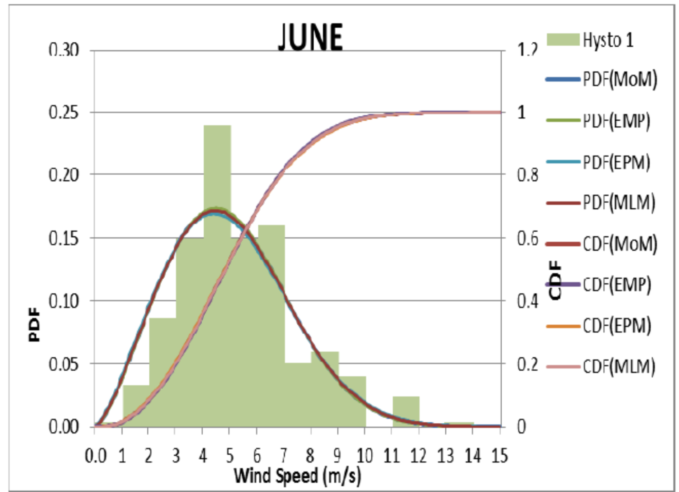

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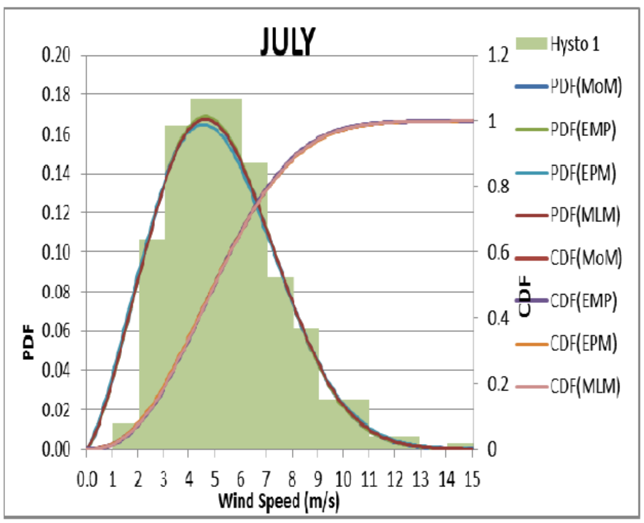

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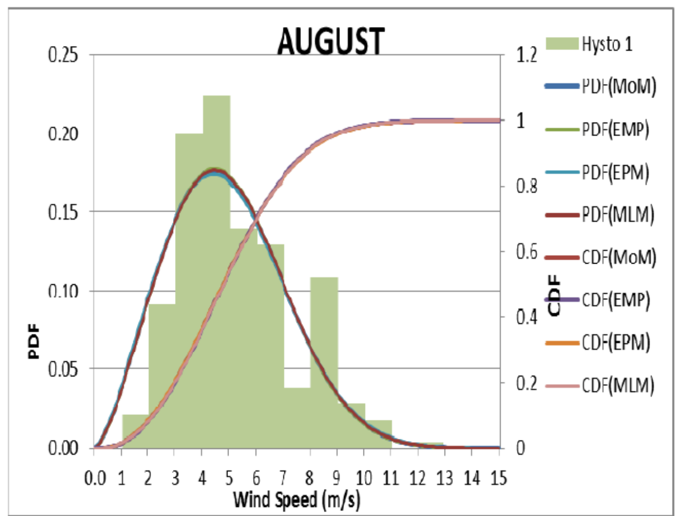

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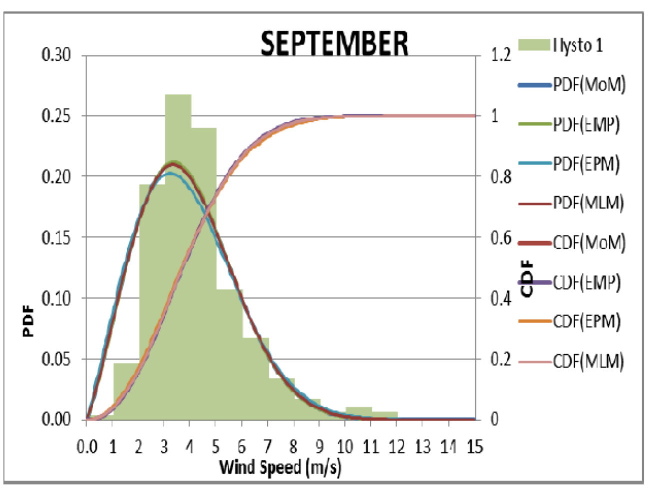

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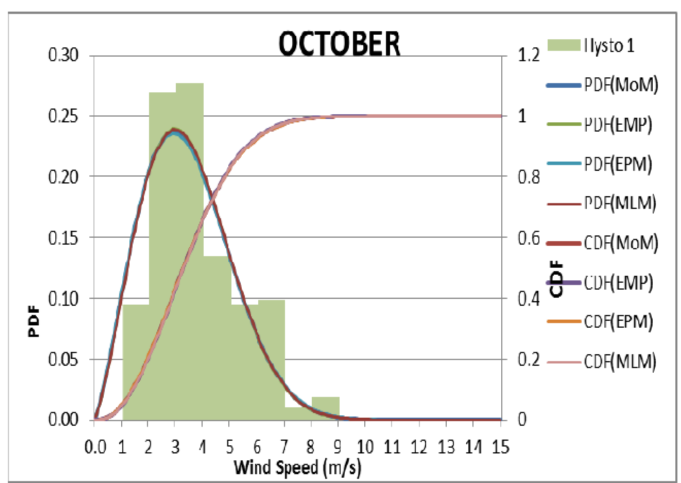

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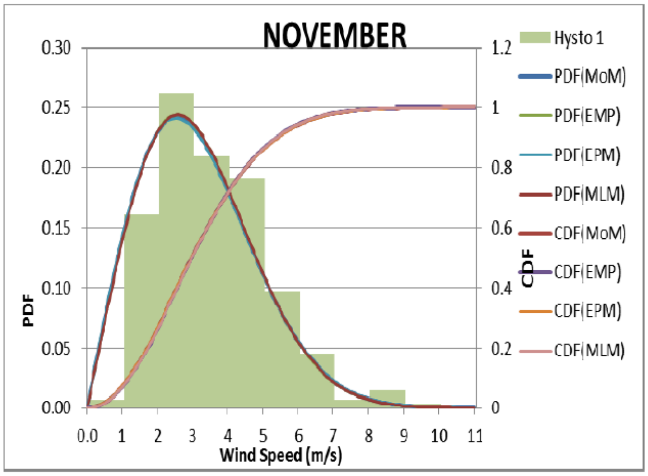

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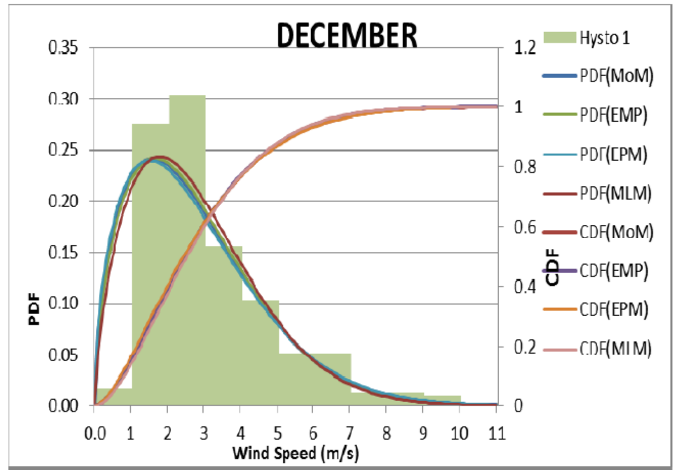

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Figure 2: Histogram of measured wind speed, PDF and CDF of wind speed distribution obtained by MoM, EMP, EPM, and MLM.

Daily Wind data at 12:00 hours for Jiwani town from 1998-2007 was used in this study to fit Weibull distribution and estimate corresponding parameters. Four methods discussed above were employed to find these parameters. The mean speed of wind of each month for a period of 10 years was also calculated, which indicates the wind potential at Jiwani station is high from April to August. The monthly mean wind speed averaged over 10 years is given in Table **1**. The estimated wind speed by the four methods is also given in the table. The Table **1** also shows the absolute deviation of estimated wind speed from measured mean value.

The histogram is generated for 12:00 hours wind speed data (1998-2007). The theoretical probability distribution and the cumulative distribution functions were generated with the help of shape and scale parameters (k and c) obtained by MoM, EMP, EPM and MLM. Both PDF and CDF are plotted along with the histogram for January to December (see Figure **2**). A comparison of shape parameter (k) and scale parameter (c) by MoM, EMP, EPM and MLM, is given in Table **2**.

CONCLUSION

The method of moments, the maximum likelihood method, the empirical method and the energy pattern factor method have been employed to calculate the parameters ‘k’ and ‘c’ of the distribution function. The wind speed data for Jiwani from 1998 to 2007 has been used to investigate the accuracy of the four models for determining of Weibull parameters.

(i) All four methods are suitable for the estimation of scale parameter of the Weibull distribution and give identical values.

(ii) There is a slight difference in the estimated value of shape parameter by using four methods, nevertheless, the difference is within the margin of ca. 0.1.

(iii) It is found that the estimated values of scale parameter determined by the four methods is slightly more than the observed mean wind speed.

(iv) The comparison of four methods in terms of estimation of wind energy suggests that the Maximum Likelihood Method is relatively in a better agreement with the measured mean wind speed.

(v) The empirical method is considered as a special case of the method of moments. The values of k and c found by these methods are very close; also the predicted mean wind speeds are in good agreement with each other. A good overlap is observed for their Weibull distribution curves.

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