Fourier Coefficients of a Class of Eta Quotients of Weight 14 with Level 12

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Abstract: Recently, Williams [1] and then Yao, Xia and Jin [2] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of $\sigma(n),\sigma(\frac{n}{2}),\sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$ and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of $\sigma_3(n)$, $\sigma_3\left(\frac{n}{2}\right),\sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. Here, by using the method of proof of Williams, we will express the even Fourier coefficients of 196 eta quotients i.e., the Fourier coefficients of the sum, f(q)+f(-q), of 196 eta quotients in terms of $\sigma_{13}(n)$, $\sigma_{13}\left(\frac{n}{2}\right)$, $\sigma_{13}\left(\frac{n}{3}\right)$, $\sigma_{13}\left(\frac{n}{4}\right)$, $\sigma_{13}\left(\frac{n}{6}\right)$ and $\sigma_{13}\left(\frac{n}{12}\right)$.

Keywords: Dedekind eta function, eta quotients, Fourier series.

1. INTRODUCTION

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\sigma_i(n):=\sum_{d \text{ positive integer}, d|n} d^i \text{, if } n \text{ is a positive integer, and} \tag{1}$$

$\sigma_i(n):=0$ if n is not a positive integer.

The Dedekind eta function is defined by

$$\eta(z):=q^{1/24}\prod_{n=1}^{\infty}(1-q^n), \tag{2}$$

where

$$q:=e^{2\pi iz}, z\in H=\{x+iy: y>0\} \tag{3}$$

and an eta quotient of level n is defined by

$$f(z):=\prod_{m|n}\eta(mz)^{a_m}, n,m\in\mathbb{N}. \tag{4}$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of eta quotients since they are the building blocks of modular forms of level n and weight k. The book of Köhler [3] (Chapter 3, pg. 39) describes such expansions by means of Hecke Theta series and develops algorithms for the determination of suitable eta quotients. One can find more information in [4-8]. We have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [9-14].

Recently, Williams, see [1] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of $\sigma(n),\sigma\left(\frac{n}{2}\right),\sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)}$$

gives the expansion found by Williams.

Then Yao, Xia and Jin [2] expressed the even Fourier coefficients of 104 eta quotients in terms of $\sigma_3(n)$, $\sigma_3(\frac{n}{2})$, $\sigma_3(\frac{n}{3})$ and $\sigma_3\left(\frac{n}{6}\right)$ One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)},$$

where the even coefficients are obtained. Motivated by these two results, we find that we can express the even Fourier coefficients of 196 eta quotients in terms of $\sigma_{13}(n)$, $\sigma_{13}\left(\frac{n}{2}\right)$, $\sigma_{13}\left(\frac{n}{3}\right)$, $\sigma_{13}\left(\frac{n}{4}\right)$, $\sigma_{13}\left(\frac{n}{6}\right)$ and $\sigma_{13}\left(\frac{n}{12}\right)$ see Table **3**. One example is as follows:

$$\frac{\eta^6(4z)\eta^{14}(6z)\eta^{14}(12z)}{\eta^6(2z)}$$

where the even coefficients are obtained. We see that the odd Fourier coefficients of 263 eta quotients are zero and even coefficients can be expressed by simple formula.

Let

$$f_1=\frac{\eta^{19}(4z)\eta^{13}(6z)\eta^7(12z)}{\eta^{11}(2z)},$$

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Table 1:

$$c_1 := -28k_0 + 2k_1$$

$$c_2 := 229824k_0 - 16440k_1 + 4k_2$$

$$\begin{aligned}c_3 :=& -\frac{28}{729}k_0 + \frac{2}{27}k_1 - \frac{104}{729}k_2 + \frac{200}{729}k_3 - \frac{128}{243}k_4 + \frac{736}{729}k_5 \\ &- \frac{1408}{729}k_6 + \frac{896}{243}k_7 - \frac{5120}{729}k_8 + \frac{9728}{729}k_9 - \frac{2048}{81}k_{10} + \frac{34816}{729}k_{11} \\ &- \frac{65536}{729}k_{12} + \frac{40960}{243}k_{13} - \frac{229376}{729}k_{14} + \frac{425984}{729}k_{15} - \frac{262144}{243}k_{16} \\ &+ \frac{1441792}{729}k_{17} - \frac{2621440}{729}k_{18} + \frac{524288}{81}k_{19} - \frac{8388608}{729}k_{20} \\ &+ \frac{14680064}{729}k_{21} - \frac{8388608}{243}k_{22} + \frac{41943040}{729}k_{23} - \frac{67108864}{729}k_{24} \\ &+ \frac{33554432}{243}k_{25} - \frac{134217728}{729}k_{26} + \frac{134217728}{729}k_{27},\end{aligned}$$

$$\begin{aligned}c_4 :=& -\frac{49551454820433920}{6973569531}k_0 + \frac{6266429034102784}{6973569531}k_1 \\ &- \frac{448223759630336}{6973569531}k_2 - \frac{4397938884608}{6973569531}k_3 + \frac{2198951723008}{6973569531}k_4 \\ &- \frac{1099475664896}{6973569531}k_5 + \frac{549755944960}{6973569531}k_6 - \frac{274913607680}{6973569531}k_7 \\ &+ \frac{137510748160}{6973569531}k_8 - \frac{68826841088}{6973569531}k_9 + \frac{34503196672}{6973569531}k_{10} \\ &- \frac{17358897152}{6973569531}k_{11} + \frac{8805056512}{6973569531}k_{12} - \frac{4545658880}{6973569531}k_{13} \\ &+ \frac{2434269184}{6973569531}k_{14} - \frac{1396097024}{6973569531}k_{15} + \frac{895320064}{6973569531}k_{16} \\ &- \frac{662454272}{6973569531}k_{17} + \frac{564330496}{6973569531}k_{18} - \frac{532791296}{6973569531}k_{19} \\ &+ \frac{535330816}{6973569531}k_{20} - \frac{554123264}{6973569531}k_{21} + \frac{581828608}{6973569531}k_{22} \\ &- \frac{613203968}{6973569531}k_{23} + \frac{647200768}{6973569531}k_{24} - \frac{681721856}{6973569531}k_{25} \\ &+ \frac{717291520}{6973569531}k_{26} - \frac{788430848}{6973569531}k_{27} + \frac{1576861696}{6973569531}k_{28},\end{aligned}$$

$$\begin{aligned}c_6 :=& -\frac{27643868972579776}{38082663208791}k_0 - \frac{18265031719974544}{38082663208791}k_1 \\ &+ \frac{44363961777336764}{38082663208791}k_2 - \frac{85773378305765560}{38082663208791}k_3 \\ &+ \frac{164466964470793856}{38082663208791}k_4 - \frac{315\,064\,929\,780\,258\,592}{38\,082\,663\,208\,791}k_5 \\ &+ \frac{602622844994339456}{38082663208791}k_6 - \frac{1150404280197361024}{38082663208791}k_7 \\ &+ \frac{2191240614215330816}{38082663208791}k_8 - \frac{4163402772772616704}{38082663208791}k_9 \\ &+ \frac{7888648788848556032}{38082663208791}k_{10} - \frac{14900927014148896768}{38082663208791}k_{11} \\ &+ \frac{28048998684929294336}{38082663208791}k_{12} - \frac{52592115320057307136}{38082663208791}k_{13} \\ &+ \frac{98172238040321589248}{38082663208791}k_{14} - \frac{182320205248569966592}{38082663208791}k_{15} \\ &+ \frac{336591526070627926016}{38082663208791}k_{16} - \frac{617084883397193433088}{38082663208791}k_{17} \\ &+ \frac{1121972972287157141504}{38082663208791}k_{18} - \frac{2019551841412983095296}{38082663208791}k_{19} \\ &+ \frac{3590314905228818579456}{38082663208791}k_{20} - \frac{6283051626861316341760}{38082663208791}k_{21}\end{aligned}$$

(Table 1). Continued.

$$+\frac{1077094620100046520 3200}{38082663208791}k_{22}-\frac{17951577553899585863680}{38082663208791}k_{23}$$

$$+\frac{28722524611812001120256}{38082663208791}k_{24}-\frac{43083787374737709924352}{38082663208791}k_{25}$$

$$+\frac{57445050137663419252736}{38082663208791}k_{26}-\frac{57445050023408491626496}{38082663208791}k_{27}$$

$$-\frac{2513608401879040}{38082663208791}k_{28},$$

$$c_{12}:=\frac{261875982447426469888}{38082663208791}k_0-\frac{111922346422090 79296}{12694221069597}k_1$$

$$+\frac{240323909182875 2384}{38082663208791}k_2+\frac{1097800746360 34048}{38082663208791}k_3$$

$$-\frac{58818459942158336}{12694221069597}k_4+\frac{321030718045896704}{38082663208791}k_5$$

$$-\frac{605551508862992384}{38082663208791}k_6+\frac{1579924778303488}{52239592879}k_7$$

$$-\frac{2191724093695492096}{38082663208791}k_8+\frac{4163270449392271360}{38082663208791}k_9$$

$$-\frac{2629291443543212032}{12694221069597}k_{10}+\frac{14899203037420568576}{38082663208791}k_{11}$$

$$-\frac{28045623195383988224}{38082663208791}k_{12}+\frac{17528573647575826432}{12694221069597}k_{13}$$

$$-\frac{98160268825009389568}{38082663208791}k_{14}+\frac{182297959641922846720}{38082663208791}k_{15}$$

$$-\frac{37394494230273753088}{4231407023199}k_{16}+\frac{617009568387760013312}{38082663208791}k_{17}$$

$$-\frac{1121836032410609254400}{38082663208791}k_{18}+\frac{673101782332504752128}{12694221069597}k_{19}$$

$$-\frac{3589876690685518643200}{38082663208791}k_{20}+\frac{6282284749320585822208}{38082663208791}k_{21}$$

$$-\frac{3589877183925868888064}{12694221069597}k_{22}+\frac{17949386469914585120768}{38082663208791}k_{23}$$

$$-\frac{28719018875612432334848}{38082663208791}k_{24}+\frac{4786503196539966078976}{4231407023199}k_{25}$$

$$-\frac{57438038662112683884544}{38082663208791}k_{26}+\frac{57438038548246248128512}{38082663208791}k_{27}$$

$$+\frac{834999053385728}{12694221069597}k_{28},$$

$$r_1:=\frac{603074757810297856}{2187}k_0-\frac{1595313830955520}{81}k_1-\frac{5319060719104}{2187}k_2$$

$$+\frac{9821099051008}{2187}k_3-\frac{6259152849920}{729}k_4+\frac{35979728478464}{2187}k_5$$

$$-\frac{68832883985408}{2187}k_6+\frac{43804286225408}{729}k_7-\frac{250316283854848}{2187}k_8$$

$$+\frac{475608968495104}{2187}k_9-\frac{100129412071424}{243}k_{10}+\frac{1702214924705792}{2187}k_{11}$$

$$-\frac{3204189765042176}{2187}k_{12}+\frac{2002627930759168}{729}k_{13}-\frac{11214754238021632}{2187}k_{14}$$

$$+\frac{20827451243044864}{2187}k_{15}-\frac{12816915260735488}{729}k_{16}$$

$$+\frac{70493120395206656}{2187}k_{17}$$

$$-\frac{128169420405555200}{2187}k_{18}+\frac{25633899541274624}{243}k_{19}$$

$$-\frac{410142564512432128}{2187}k_{20}$$

$$+\frac{717749695611363328}{2187}k_{21}-\frac{410142764638945280}{729}k_{22}$$

$$+\frac{2050714099643064320}{2187}k_{23}$$

(Table 1). Continued.

$$-\frac{3281142853004263424}{2187}k_{24}+\frac{1640571518000144384}{729}k_{25}$$
$$-\frac{6562286254996602880}{2187}k_{26}+\frac{6562286254996602880}{2187}k_{27},$$

$$r_2:=-\frac{498118393672297600}{2187}k_0+\frac{1317673389068992}{81}k_1+\frac{4178113615936}{2187}k_2$$
$$-\frac{7650455291008}{2187}k_3+\frac{4872771276416}{729}k_4-\frac{28009307999648}{2187}k_5$$
$$+\frac{53583642658304}{2187}k_6-\frac{34099440250880}{729}k_7+\frac{194857366070272}{2187}k_8$$
$$-\frac{370234081569280}{2187}k_9+\frac{77944846942208}{243}k_{10}-\frac{1325073222729728}{2187}k_{11}$$
$$+\frac{2494270956634112}{2187}k_{12}-\frac{1558926621147136}{729}k_{13}+\frac{8730019381903360}{2187}k_{14}$$
$$-\frac{16212934574866432}{2187}k_{15}+\frac{9977209092308992}{729}k_{16}-\frac{54874723778428928}{2187}k_{17}$$
$$+\frac{99772320966508544}{2187}k_{18}-\frac{19954477794590720}{243}k_{19}$$
$$+\frac{319271797805547520}{2187}k_{20}$$
$$-\frac{558725833217277952}{2187}k_{21}+\frac{319271978714267648}{729}k_{22}$$
$$-\frac{1596360146806636544}{2187}k_{23}$$
$$+\frac{2554176505433292800}{2187}k_{24}-\frac{1277088337340923904}{729}k_{25}$$
$$+\frac{5108353518612250624}{2187}k_{26}-\frac{5108353518612250624}{2187}k_{27},$$

$$r_3:=-\frac{34649292590809088}{243}k_0+\frac{91637694595072}{9}k_1+\frac{1374934532096}{243}k_2$$
$$-\frac{2615544774656}{243}k_3+\frac{1672004501504}{81}k_4-\frac{9613118144512}{243}k_5$$
$$+\frac{18390648094720}{243}k_6-\frac{11703468818432}{81}k_7+\frac{66878587535360}{243}k_8$$
$$-\frac{127071673253888}{243}k_9+\frac{26752291438592}{27}k_{10}-\frac{454793282584576}{243}k_{11}$$
$$+\frac{856087134994432}{243}k_{12}-\frac{535056887971840}{81}k_{13}+\frac{2996327809286144}{243}k_{14}$$
$$-\frac{5564620332204032}{243}k_{15}+\frac{3424386484535296}{81}k_{16}-\frac{18834142949146624}{243}k_{17}$$
$$+\frac{34243916973998080}{243}k_{18}-\frac{6848786109169664}{27}k_{19}$$
$$+\frac{109580606107811840}{243}k_{20}$$
$$-\frac{191766093000998912}{243}k_{21}+\frac{109580636556886016}{81}k_{22}$$
$$-\frac{547903221482782720}{243}k_{23}$$
$$+\frac{876645193803366400}{243}k_{24}-\frac{438322608877469696}{81}k_{25}$$
$$+\frac{1753290459461451776}{243}k_{26}-\frac{1753290459461451776}{243}k_{27},$$

$$r_4:=\frac{25339840443904}{729}k_0-\frac{67035237632}{27}k_1-\frac{11295232}{729}k_2+\frac{2032768}{729}k_3$$
$$-\frac{91648}{243}k_4+\frac{13760}{729}k_5+\frac{18304}{729}k_6-\frac{11648}{243}k_7+\frac{66560}{729}k_8-\frac{126464}{729}k_9$$
$$+\frac{26624}{81}k_{10}-\frac{452608}{729}k_{11}+\frac{851968}{729}k_{12}-\frac{532480}{243}k_{13}+\frac{2981888}{729}k_{14}$$

(Table 1). Continued.

$$-\frac{5537792}{729}k_{15}+\frac{3407872}{243}k_{16}-\frac{18743296}{729}k_{17}+\frac{34078720}{729}k_{18}-\frac{6815744}{81}k_{19}+\frac{109051904}{729}k_{20}-\frac{190840832}{729}k_{21}+\frac{109051904}{243}k_{22}-\frac{545259520}{729}k_{23}+\frac{872415232}{729}k_{24}-\frac{436207616}{243}k_{25}+\frac{1744830464}{729}k_{26}-\frac{1744830464}{729}k_{27},$$

$$r_5:=\frac{629995847936}{243}k_0-\frac{2469574016}{9}k_1+\frac{43358962816}{243}k_2-\frac{83385070720}{243}k_3+\frac{53367831296}{81}k_4-\frac{306872041088}{243}k_5+\frac{587071295360}{243}k_6-\frac{373597325696}{81}k_7+\frac{2134874002432}{243}k_8-\frac{4056313100800}{243}k_9+\frac{853970087936}{27}k_{10}-\frac{14517627416576}{243}k_{11}+\frac{27327513755648}{243}k_{12}-\frac{17079808147456}{81}k_{13}+\frac{95647443877888}{243}k_{14}-\frac{177631754420224}{243}k_{15}+\frac{109312241041408}{81}k_{16}-\frac{601219052994560}{243}k_{17}+\frac{1093128037597184}{243}k_{18}-\frac{218625996750848}{27}k_{19}+\frac{3498020763074560}{243}k_{20}-\frac{6121542765248512}{243}k_{21}+\frac{3498027197136896}{81}k_{22}-\frac{17490146119122944}{243}k_{23}+\frac{27984245225947136}{243}k_{24}-\frac{13992126303961088}{81}k_{25}+\frac{55968512597819392}{243}k_{26}-\frac{55968512597819392}{243}k_{27},$$

$$r_6:=\frac{793976503953280}{243}k_0-\frac{2100466546496}{9}k_1+\frac{955585856}{243}k_2-\frac{2191567616}{243}k_3+\frac{1430112896}{81}k_4-\frac{8235186400}{243}k_5+\frac{15750961792}{243}k_6-\frac{10021676672}{81}k_7+\frac{57262991360}{243}k_8-\frac{108797692928}{243}k_9+\frac{22904754176}{27}k_{10}-\frac{389382369280}{243}k_{11}+\frac{732958326784}{243}k_{12}-\frac{458100613120}{81}k_{13}+\frac{2565370068992}{243}k_{14}-\frac{4764266946560}{243}k_{15}+\frac{2931859849216}{81}k_{16}-\frac{16125240451072}{243}k_{17}+\frac{29318631669760}{243}k_{18}-\frac{5863727882240}{27}k_{19}+\frac{93819661156352}{243}k_{20}-\frac{164184422948864}{243}k_{21}+\frac{93819675754496}{81}k_{22}-\frac{469098395361280}{243}k_{23}+\frac{750557448503296}{243}k_{24}-\frac{375278728896512}{81}k_{25}+\frac{1501114924875776}{243}k_{26}-\frac{1501114924875776}{243}k_{27},$$

$$r_7:=-\frac{32558191232}{27}k_0+87155648k_1-\frac{55237312}{27}k_2+\frac{106230400}{27}k_3-\frac{67990400}{9}k_4+\frac{390960992}{27}k_5-\frac{747954944}{27}k_6+\frac{475989248}{9}k_7-\frac{2720035840}{27}k_8+\frac{5168243200}{27}k_9-\frac{1088086016}{3}k_{10}+\frac{18498019328}{27}k_{11}-\frac{34820784128}{27}k_{12}+\frac{21763563520}{9}k_{13}-\frac{121878937600}{27}k_{14}+\frac{226351710208}{27}k_{15}-\frac{139296243712}{9}k_{16}+\frac{766143758336}{27}k_{17}-\frac{1393012244480}{27}k_{18}+\frac{278606643200}{3}k_{19}-\frac{4457765011456}{27}k_{20}$$

(Table 1). Continued.

$$+\frac{7801176850432}{27}k_{21}-\frac{4457857286144}{9}k_{22}+\frac{22289454202880}{27}k_{23}$$
$$-\frac{35663328051200}{27}k_{24}+\frac{17831731134464}{9}k_{25}$$
$$-\frac{71327058755584}{27}k_{26}+\frac{71327058755584}{27}k_{27},$$

$$r_8:=-\frac{8435766769024}{243}k_0+\frac{22316388416}{9}k_1+\frac{3739840}{243}k_2-\frac{675712}{243}k_3$$
$$+\frac{30592}{81}k_4-\frac{4832}{243}k_5-\frac{5632}{243}k_6+\frac{3584}{81}k_7-\frac{20480}{243}k_8+\frac{38912}{243}k_9$$
$$-\frac{8192}{27}k_{10}+\frac{139264}{243}k_{11}-\frac{262144}{243}k_{12}+\frac{163840}{81}k_{13}-\frac{917504}{243}k_{14}$$
$$+\frac{1703936}{243}k_{15}-\frac{1048576}{81}k_{16}+\frac{5767168}{243}k_{17}-\frac{10485760}{243}k_{18}$$
$$+\frac{2097152}{27}k_{19}-\frac{33554432}{243}k_{20}+\frac{58720256}{243}k_{21}-\frac{33554432}{81}k_{22}$$
$$+\frac{167772160}{243}k_{23}-\frac{268435456}{243}k_{24}+\frac{134217728}{81}k_{25}$$
$$-\frac{536870912}{243}k_{26}+\frac{536870912}{243}k_{27},$$

$$r_9:=\frac{22853743673344}{243}k_0-\frac{62410588160}{9}k_1+\frac{106261250048}{243}k_2$$
$$-\frac{205166870528}{243}k_3+\frac{131529113600}{81}k_4-\frac{756837842944}{243}k_5$$
$$+\frac{1448317222912}{243}k_6-\frac{921786712064}{81}k_7+\frac{5267709820928}{243}k_8$$
$$-\frac{10009000607744}{243}k_9+\frac{2107199651840}{27}k_{10}-\frac{35822818754560}{243}k_{11}$$
$$+\frac{67431689420800}{243}k_{12}-\frac{42145006944256}{81}k_{13}+\frac{236012768264192}{243}k_{14}$$
$$-\frac{438310304940032}{243}k_{15}+\frac{269729767555072}{81}k_{16}-\frac{1483514955759616}{243}k_{17}$$
$$+\frac{2697301354283008}{243}k_{18}-\frac{539460453466112}{27}k_{19}+\frac{8631369108094976}{243}k_{20}$$
$$-\frac{15104897989541888}{243}k_{21}+\frac{8631371019124736}{81}k_{22}-\frac{43156857420709888}{243}k_{23}$$
$$+\frac{69050974188666880}{243}k_{24}-\frac{34525487787081728}{81}k_{25}$$
$$+\frac{138101952533823488}{243}k_{26}-\frac{138101952533823488}{243}k_{27},$$

$$r_{10}:=\frac{727936359783268352}{2187}k_0-\frac{1925742066532352}{81}k_1-\frac{243560480768}{2187}k_2$$
$$+\frac{120676745216}{2187}k_3-\frac{19406946304}{729}k_4+\frac{25054314496}{2187}k_5$$
$$-\frac{4772823040}{2187}k_6-\frac{4135485440}{729}k_7+\frac{34357411840}{2187}k_8$$
$$-\frac{70623920128}{2187}k_9+\frac{15164014592}{243}k_{10}-\frac{259113844736}{2187}k_{11}$$
$$+\frac{488403402752}{2187}k_{12}-\frac{305361485824}{729}k_{13}+\frac{1710187380736}{2187}k_{14}$$
$$-\frac{3176143224832}{2187}k_{15}+\frac{1954563063808}{729}k_{16}-\frac{10750116724736}{2187}k_{17}$$
$$+\frac{19545676611584}{2187}k_{18}-\frac{3909135859712}{243}k_{19}+\frac{62546176147456}{2187}k_{20}$$
$$-\frac{109455809413120}{2187}k_{21}+\frac{62546176999424}{729}k_{22}-\frac{312730885259264}{2187}k_{23}$$

(Table 1). Continued.

$$+\frac{500369416552448}{2187}k_{24}-\frac{250184708292608}{729}k_{25}$$
$$+\frac{1000738833203200}{2187}k_{26}-\frac{1000738833203200}{2187}k_{27},$$

$$r_{11}:=-\frac{228591632384}{243}k_0+\frac{626753536}{9}k_1-\frac{1175093248}{243}k_2$$
$$+\frac{2246778880}{243}k_3-\frac{1434222592}{81}k_4+\frac{8237490176}{243}k_5$$
$$-\frac{15751233536}{243}k_6+\frac{10021642240}{81}k_7-\frac{57262735360}{243}k_8$$
$$+\frac{108797206528}{243}k_9-\frac{22904651776}{27}k_{10}+\frac{389380628480}{243}k_{11}$$
$$-\frac{732955049984}{243}k_{12}+\frac{458098565120}{81}k_{13}-\frac{2565358600192}{243}k_{14}$$
$$+\frac{4764245647360}{243}k_{15}-\frac{2931846742016}{81}k_{16}+\frac{16125168361472}{243}k_{17}$$
$$-\frac{29318500597760}{243}k_{18}+\frac{5863701667840}{27}k_{19}-\frac{93819241725952}{243}k_{20}$$
$$+\frac{164183688945664}{243}k_{21}-\frac{93819256324096}{81}k_{22}+\frac{469096298209280}{243}k_{23}$$
$$-\frac{750554093060096}{243}k_{24}+\frac{375277051174912}{81}k_{25}$$
$$-\frac{1501108213989376}{243}k_{26}+\frac{1501108213989376}{243}k_{27},$$

$$r_{12}:=\frac{1142958161920}{729}k_0-\frac{3131998208}{27}k_1+\frac{5827690496}{729}k_2$$
$$-\frac{11190099968}{729}k_3+\frac{7157841920}{243}k_4-\frac{41151619072}{729}k_5$$
$$+\frac{78724317184}{729}k_6-\frac{50098921472}{243}k_7+\frac{286289788928}{729}k_8$$
$$-\frac{543966126080}{729}k_9+\frac{114521489408}{81}k_{10}-\frac{1946891198464}{729}k_{11}$$
$$+\frac{3664767287296}{729}k_{12}-\frac{2290491498496}{729}k_{13}+\frac{12826793000960}{729}k_{14}$$
$$-\frac{23821232218112}{729}k_{15}+\frac{14659236364288}{243}k_{16}-\frac{80625853751296}{729}k_{17}$$
$$+\frac{146592518914048}{729}k_{18}-\frac{29318510551040}{81}k_{19}+\frac{469096232517632}{729}k_{20}$$
$$-\frac{820918472597504}{729}k_{21}+\frac{469096292237312}{243}k_{22}-\frac{2345481526878208}{729}k_{23}$$
$$+\frac{3752770505113600}{729}k_{24}-\frac{1876385270472704}{243}k_{25}$$
$$+\frac{7505541117722624}{729}k_{26}-\frac{7505541117722624}{729}k_{27},$$

$$r_{13}:=\frac{65445517307314501184}{2324523177}k_0-\frac{8393952427404604960}{2324523177}k_1$$
$$+\frac{607510207588656032}{2324523177}k_2$$
$$+\frac{9576900154769312}{2324523177}k_3-\frac{5371838447097280}{2324523177}k_4+\frac{2902824593746784}{2324523177}k_5$$
$$-\frac{1537487675124928}{2324523177}k_6+\frac{810570917187968}{2324523177}k_7-\frac{431764857706240}{2324523177}k_8$$
$$+\frac{235417477283840}{2324523177}k_9-\frac{132235585650688}{2324523177}k_{10}+\frac{75931831402496}{2324523177}k_{11}$$
$$-\frac{43069291823104}{2324523177}k_{12}+\frac{21925213601792}{2324523177}k_{13}-\frac{6642512035840}{2324523177}k_{14}$$
$$-\frac{5711647178752}{2324523177}k_{15}+\frac{16599389241344}{2324523177}k_{16}-\frac{26756068704256}{2324523177}k_{17}$$

(Table 1). Continued.

$$+\frac{36545070891008}{2324523177}k_{18}-\frac{46152380416000}{2324523177}k_{19}+\frac{55666697633792}{2324523177}k_{20}$$
$$-\frac{65136664674304}{2324523177}k_{21}+\frac{74582310649856}{2324523177}k_{22}-\frac{84017942069248}{2324523177}k_{23}$$
$$+\frac{93446420234240}{2324523177}k_{24}-\frac{102873467748352}{2324523177}k_{25}+\frac{112297653960704}{2324523177}k_{26}$$
$$-\frac{131146026385408}{2324523177}k_{27}+\frac{262292052770816}{2324523177}k_{28},$$

$$r_{14}:=\frac{602980136368654566504393728}{12694221069597}k_0-\frac{773616725769691966649760256}{12694221069597}k_1$$
$$+\frac{56363440530095077101992 96}{12694221069597}k_2+\frac{55215036218874902041088}{12694221069597}k_3$$
$$-\frac{27576484252083655779328}{12694221069597}k_4+\frac{13771802569275983058944}{12694221069597}k_5$$
$$-\frac{6880059002977198133248}{12694221069597}k_6+\frac{3442116343477837758464}{12694221069597}k_7$$
$$-\frac{1727561320263769391104}{12694221069597}k_8+\frac{869672666586216660992}{12694221069597}k_9$$
$$-\frac{433471649910062055424}{12694221069597}k_{10}+\frac{199785287005158244352}{12694221069597}k_{11}$$
$$-\frac{57382989330951700480}{12694221069597}k_{12}-\frac{51034154345422913536}{12694221069597}k_{13}$$
$$+\frac{155759696565878128640}{12694221069597}k_{14}-\frac{273624030563958784000}{12694221069597}k_{15}$$
$$+\frac{414686449881917161472}{12694221069597}k_{16}-\frac{585660218253467975680}{12694221069597}k_{17}$$
$$+\frac{791546064470948446208}{12694221069597}k_{18}-\frac{1036527969893115756544}{12694221069597}k_{19}$$
$$+\frac{1324342022124828360704}{12694221069597}k_{20}-\frac{1658539881931104059392}{12694221069597}k_{21}$$
$$+\frac{2042541476619931615232}{12694221069597}k_{22}-\frac{2479740386807893983232}{12694221069597}k_{23}$$
$$+\frac{2973477499728762503168}{12694221069597}k_{24}-\frac{3530434589850079854592}{12694221069597}k_{25}$$
$$+\frac{4213831634372294868992}{12694221069597}k_{26}-\frac{5580625723416724897792}{12694221069597}k_{27}$$
$$+\frac{11161251446833449795584}{12694221069597}k_{28},$$

$$r_{15}:=-\frac{439725956648625960135887948 8}{12694221069597}k_0$$
$$+\frac{564163782325811947435372928}{12694221069597}k_1$$
$$-\frac{41103040268493150606014848}{12694221069597}k_2-\frac{40307918166302664784806 4}{12694221069597}k_3+$$
$$\frac{20147770523307134372480 0}{12694221069597}k_4-\frac{10070360599026377341081 6}{12694221069597}k_5$$
$$+\frac{5033779411981844233932 8}{12694221069597}k_6-\frac{2517029368510318221721 6}{12694221069597}k_7$$
$$+\frac{1259500494218941536665 6}{12694221069597}k_8-\frac{630690418502151739801 6}{12694221069597}k_9$$
$$+\frac{315134249811183337472 0}{12694221069597}k_{10}-\frac{154814741478546354995 2}{12694221069597}k_{11}$$
$$+\frac{70425721133191056588 8}{12694221069597}k_{12}-\frac{219461088492836356096}{12694221069597}k_{13}$$
$$-\frac{110153772205992706048}{12694221069597}k_{14}+\frac{391055645876799930368}{12694221069597}k_{15}$$
$$-\frac{681118017392778674176}{12694221069597}k_{16}+\frac{1014621423537649221632}{12694221069597}k_{17}$$

(Table 1). Continued.

$$-\frac{1414177408492370526208}{12694221069597}k_{18}+\frac{1897267467166562779136}{12694221069597}k_{19}$$
$$-\frac{2478935551276151996416}{12694221069597}k_{20}+\frac{3173711287970259009536}{12694221069597}k_{21}$$
$$-\frac{3995994623556577263616}{12694221069597}k_{22}+\frac{4960825110952128413696}{12694221069597}k_{23}$$
$$-\frac{6084521516292590534656}{12694221069597}k_{24}+\frac{7399721371849319776256}{12694221069597}k_{25}$$
$$-\frac{9097928127838583259136}{12694221069597}k_{26}+\frac{12494341639817110224896}{12694221069597}k_{27}$$
$$-\frac{24988683279634220449792}{12694221069597}k_{28},$$

$$r_{16}:=\frac{383266020740757471145003107 2}{12694221069597}k_0$$
$$-\frac{4917260892635543351730878 56}{12694221069597}k_1$$
$$+\frac{3582528119701095776141182 4}{12694221069597}k_2+\frac{3515019529078319007074 72}{12694221069597}k_3$$
$$-\frac{1757419947111599390186 56}{12694221069597}k_4+\frac{8786645771215236297718 4}{12694221069597}k_5$$
$$-\frac{4393299989775905235155 2}{12694221069597}k_6+\frac{2196939568300132395046 4}{12694221069597}k_7$$
$$-\frac{1098906654252440492416 0}{12694221069597}k_8+\frac{549789504869740798976 0}{12694221069597}k_9$$
$$-\frac{2748014822046977357824}{12694221069597}k_{10}+\frac{1364154705457278801920}{12694221069597}k_{11}$$
$$-\frac{657267814097180667904}{12694221069597}k_{12}+\frac{280803787376069795840}{12694221069597}k_{13}$$
$$-\frac{59318933767062077440}{12694221069597}k_{14}-\frac{97738231775011962880}{12694221069597}k_{15}$$
$$+\frac{238637472275158728704}{12694221069597}k_{16}-\frac{391175397479655276544}{12694221069597}k_{17}$$
$$+\frac{573076582196137558016}{12694221069597}k_{18}-\frac{797697612454331809792}{12694221069597}k_{19}$$
$$+\frac{1076376263515850276864}{12694221069597}k_{20}-\frac{1420110217789893640192}{12694221069597}k_{21}$$
$$+\frac{1839894862560766853120}{12694221069597}k_{22}-\frac{2347398885442910814208}{12694221069597}k_{23}$$
$$+\frac{2955637574711038312448}{12694221069597}k_{24}-\frac{3690641506914574925824}{12694221069597}k_{25}$$
$$+\frac{4679175924988929769472}{12694221069597}k_{26}-\frac{6656244761137639456768}{12694221069597}k_{27}$$
$$+\frac{13312489522275278913536}{12694221069597}k_{28},$$

$$r_{17}:=-\frac{9555763511220197734352}{2324523177}k_0+\frac{12259937740059255617 68}{2324523177}k_1$$
$$-\frac{8932110340467686813 6}{2324523177}k_2$$
$$-\frac{876466706267647352}{2324523177}k_3+\frac{438229821811153648}{2324523177}k_4$$
$$-\frac{219114871646975456}{2324523177}k_5$$
$$+\frac{109561045318986688}{2324523177}k_6-\frac{54787624104269696}{2324523177}k_7+\frac{27404562021428992}{2324523177}k_8$$
$$-\frac{13716522470120960}{2324523177}k_9+\frac{6876150300654592}{2324523177}k_{10}-\frac{3459453869373440}{2324523177}k_{11}$$
$$+\frac{1754749586599936}{2324523177}k_{12}-\frac{905879752024064}{2324523177}k_{13}+\frac{485074074320896}{2324523177}k_{14}$$

(Table 1). Continued.

$$- \frac{278124155715584}{2324523177} k_{15} + \frac{178219662868480}{2324523177} k_{16} - \frac{131602790432768}{2324523177} k_{17}$$
$$+ \frac{111629728153600}{2324523177} k_{18} - \frac{104508385968128}{2324523177} k_{19} + \frac{103342718746624}{2324523177} k_{20}$$
$$- \frac{103744333955072}{2324523177} k_{21} + \frac{102578665160704}{2324523177} k_{22} - \frac{95457319043072}{2324523177} k_{23}$$
$$+ \frac{75484248506368}{2324523177} k_{24} - \frac{28867359358976}{2324523177} k_{25} - \frac{71037167009792}{2324523177} k_{26}$$
$$+ \frac{270846219747328}{2324523177} k_{27} - \frac{541692439494656}{2324523177} k_{28},$$

$$r_{18} := - \frac{115880789627371 52}{6973569531} k_0 + \frac{1166714887340032}{6973569531} k_1$$
$$+ \frac{230708949680128}{6973569531} k_2$$
$$- \frac{315300633509888}{6973569531} k_3 + \frac{286187145920512}{6973569531} k_4 - \frac{257348536107008}{6973569531} k_5$$
$$+ \frac{228647365378048}{6973569531} k_6 - \frac{200014913994752}{6973569531} k_7 + \frac{171416822480896}{6973569531} k_8$$
$$- \frac{142835910705152}{6973569531} k_9 + \frac{114263588995072}{6973569531} k_{10} - \frac{85695562121216}{6973569531} k_{11}$$
$$+ \frac{57129682862080}{6973569531} k_{12} - \frac{28564877213696}{6973569531} k_{13} + \frac{608567296}{6973569531} k_{14}$$
$$+ \frac{28563391774720}{6973569531} k_{15} - \frac{57127257767936}{6973569531} k_{16} + \frac{85691056783360}{6973569531} k_{17}$$
$$- \frac{114254822113280}{6973569531} k_{18} + \frac{142818570797056}{6973569531} k_{19} - \frac{171382310961152}{6973569531} k_{20}$$
$$+ \frac{199946047062016}{6973569531} k_{21} - \frac{228509780934656}{6973569531} k_{22} + \frac{257073513889792}{6973569531} k_{23}$$
$$- \frac{285637246189568}{6973569531} k_{24} + \frac{314200978358272}{6973569531} k_{25} - \frac{342764710264832}{6973569531} k_{26}$$
$$+ \frac{399892174077952}{6973569531} k_{27} - \frac{799784348155904}{6973569531} k_{28},$$

$$r_{19} := \frac{9393124352834789075319616912}{12694221069597} k_0$$
$$- \frac{1205127353834611636881555080}{12694221069597} k_1$$
$$+ \frac{878008595536092662325562 96}{12694221069597} k_2 + \frac{861547678368538125520408}{12694221069597} k_3$$
$$- \frac{430769937002844595723568}{12694221069597} k_4 + \frac{215384598731625292508896}{12694221069597} k_5$$
$$- \frac{107695527561807207575360}{12694221069597} k_6 + \frac{53854292338791349673728}{12694221069597} k_7$$
$$- \frac{26936898773352911576576}{12694221069597} k_8 + \frac{13480906437520486389760}{12694221069597} k_9$$
$$- \frac{6755195963867300956160}{12694221069597} k_{10} + \frac{3393586871841028630528}{12694221069597} k_{11}$$
$$- \frac{1712833857242674221056}{12694221069597} k_{12} + \frac{870333574608036683776}{12694221069597} k_{13}$$
$$- \frac{444136974672224043008}{12694221069597} k_{14} + \frac{221635685718737010688}{12694221069597} k_{15}$$
$$- \frac{95039797683344408576}{12694221069597} k_{16} + \frac{77807583398852853 76}{12694221069597} k_{17}$$
$$+ \frac{71159004378669383680}{12694221069597} k_{18} - \frac{161417106605792116736}{12694221069597} k_{19}$$
$$+ \frac{277208225465967345664}{12694221069597} k_{20} - \frac{431637801002964598784}{12694221069597} k_{21}$$
$$+ \frac{637729108115521208320}{12694221069597} k_{22} - \frac{910477400428707725312}{12694221069597} k_{23}$$

(Table 1). Continued.

$$+\frac{1268821889020261138432}{12694221069597}k_{24}-\frac{1750640996045655425024}{12694221069597}k_{25}$$
$$+\frac{2479409339938731458560}{12694221069597}k_{26}-\frac{3936946027724883525632}{12694221069597}k_{27}$$
$$+\frac{7873892055449767051264}{12694221069597}k_{28},$$

$$r_{20}:=-\frac{78276036798203462792467528 0}{1410469007733}k_0$$
$$+\frac{1004272799929599582426911 12}{1410469007733}k_1$$
$$-\frac{7316738185452832876531240}{1410469007733}k_2-\frac{717957696883570154716 72}{1410469007733}k_3$$
$$+\frac{3589759503453441675 2624}{1410469007733}k_4-\frac{1794879336108620559 2896}{1410469007733}k_5$$
$$+\frac{897469066329281798 6048}{1410469007733}k_6-\frac{4487923979277156740224}{1410469007733}k_7$$
$$+\frac{2244837002143469919488}{1410469007733}k_8-\frac{1123575013992080828416}{1410469007733}k_9$$
$$+\frac{563234778019505567744}{1410469007733}k_{10}-\frac{283336300956870645760}{1410469007733}k_{11}$$
$$+\frac{143660629105531105280}{1410469007733}k_{12}-\frac{74064745298089050112}{1410469007733}k_{13}$$
$$+\frac{39489658385079468032}{1410469007733}k_{14}-\frac{22358531191172546560}{1410469007733}k_{15}$$
$$+\frac{13873637278949212160}{1410469007733}k_{16}-\frac{9555231502973845504}{1410469007733}k_{17}$$
$$+\frac{7104384631683940352}{1410469007733}k_{18}-\frac{5220138909669474304}{1410469007733}k_{19}$$
$$+\frac{3097954623020761088}{1410469007733}k_{20}-\frac{633877931023237 12}{1410469007733}k_{21}$$
$$-\frac{4523769872821190656}{1410469007733}k_{22}+\frac{11461211080333180928}{1410469007733}k_{23}$$
$$-\frac{21861597360713138176}{1410469007733}k_{24}+\frac{37950251776520044544}{1410469007733}k_{25}$$
$$-\frac{65415442463180849152}{1410469007733}k_{26}+\frac{120345823836502458368}{1410469007733}k_{27}$$
$$-\frac{240691647673004916736}{1410469007733}k_{28},$$

$$r_{21}:=\frac{24204370597605104}{6973569531}k_0-\frac{3122018638296760}{6973569531}k_1+\frac{229811561919416}{6973569531}k_2$$
$$+\frac{3003224978408}{6973569531}k_3-\frac{2087374610512}{6973569531}k_4+\frac{1099475664896}{6973569531}k_5$$
$$-\frac{549755944960}{6973569531}k_6$$
$$+\frac{274913607680}{6973569531}k_7-\frac{137510748160}{6973569531}k_8+\frac{68826841088}{6973569531}k_9-\frac{34503196672}{6973569531}k_{10}$$
$$+\frac{17358897152}{6973569531}k_{11}-\frac{8805056512}{6973569531}k_{12}+\frac{4545658880}{6973569531}k_{13}-\frac{2434269184}{6973569531}k_{14}$$
$$+\frac{1396097024}{6973569531}k_{15}-\frac{895320064}{6973569531}k_{16}+\frac{662454272}{6973569531}k_{17}-\frac{564330496}{6973569531}k_{18}$$
$$+\frac{532791296}{6973569531}k_{19}-\frac{535330816}{6973569531}k_{20}+\frac{554123264}{6973569531}k_{21}-\frac{581828608}{6973569531}k_{22}$$
$$+\frac{613203968}{6973569531}k_{23}-\frac{647200768}{6973569531}k_{24}+\frac{681721856}{6973569531}k_{25}-\frac{717291520}{6973569531}k_{26}$$
$$+\frac{788430848}{6973569531}k_{27}-\frac{1576861696}{6973569531}k_{28},$$

(Table 1). Continued.

$$r_{22} := \frac{1572472491474686836736}{6973569531}k_0 - \frac{2017876754121793 20832}{6973569531}k_1 + \frac{1473788346666411 6224}{6973569531}k_2 + \frac{112069832550514688}{6973569531}k_3 - \frac{46865374365614080}{6973569531}k_4 + \frac{16805892869586944}{6973569531}k_5 - \frac{3861905656643584}{6973569531}k_6 - \frac{981380058972160}{6973569531}k_7 + \frac{2231309087277056}{6973569531}k_8 - \frac{2141605416140800}{6973569531}k_9 + \frac{1839079456636928}{6973569531}k_{10} - \frac{1887187997163520}{6973569531}k_{11} + \frac{2567607849058304}{6973569531}k_{12} - \frac{4021229000851456}{6973569531}k_{13} + \frac{6318444863946752}{6973569531}k_{14} - \frac{9494503726907392}{6973569531}k_{15} + \frac{13566978151153664}{6973569531}k_{16} - \frac{18544706000257024}{6973569531}k_{17} + \frac{24432054623141888}{6973569531}k_{18} - \frac{31231259277131776}{6973569531}k_{19} + \frac{38943386008027136}{6973569531}k_{20} - \frac{47569019421589504}{6973569531}k_{21} + \frac{57108400237838336}{6973569531}k_{22} - \frac{67561700399644672}{6973569531}k_{23} + \frac{78928954295582720}{6973569531}k_{24} - \frac{91210230702800896}{6973569531}k_{25} + \frac{105319552132579328}{6973569531}k_{26} - \frac{133538194992136192}{6973569531}k_{27} + \frac{267076389984272384}{6973569531}k_{28},$$

$$r_{23} := -\frac{2724894103212472427152}{4231407023199}k_0 + \frac{3493500218512368 99080}{4231407023199}k_1 - \frac{255568417575569834 32}{4231407023199}k_2 - \frac{93856138910854936}{4231407023199}k_3 + \frac{12108953982278192}{4231407023199}k_4 + \frac{1562054815526624}{4231407023199}k_5 - \frac{2064278326732864}{4231407023199}k_6 + \frac{1161348050665472}{4231407023199}k_7 - \frac{571381906898944}{4231407023199}k_8 + \frac{273299513655296}{4231407023199}k_9 - \frac{121162336043008}{4231407023199}k_{10} + \frac{41994425876480}{4231407023199}k_{11} + \frac{685510197248}{4231407023199}k_{12} - \frac{25124799594496}{4231407023199}k_{13} + \frac{40440425283584}{4231407023199}k_{14} - \frac{51197559488512}{4231407023199}k_{15} + \frac{59672107581440}{4231407023199}k_{16} - \frac{67008702988288}{4231407023199}k_{17} + \frac{73772981682176}{4231407023199}k_{18} - \frac{80254442389504}{4231407023199}k_{19} + \frac{86591153733632}{4231407023199}k_{20} - \frac{92858830766080}{4231407023199}k_{21} + \frac{99088650272768}{4231407023199}k_{22} - \frac{105302881386496}{4231407023199}k_{23} + \frac{111505977933824}{4231407023199}k_{24} - \frac{117706847567872}{4231407023199}k_{25} + \frac{123903263375360}{4231407023199}k_{26} - \frac{136296094990336}{4231407023199}k_{27} + \frac{272592189980672}{4231407023199}k_{28}.$$

$$f_2 = \frac{\eta^{14}(4z)\eta^{18}(6z)\eta^{6}(12z)}{\eta^{10}(2z)},$$

$$f_3 = \frac{\eta^{16}(4z)\eta^{4}(6z)\eta^{16}(12z)}{\eta^{8}(2z)},$$

$$f_4 = \frac{\eta^{12}(2z)\eta^{12}(4z)\eta^{8}(6z)}{\eta^{4}(12z)},$$

$$f_5 = \frac{\eta^{6}(4z)\eta^{14}(6z)\eta^{14}(12z)}{\eta^{6}(2z)},$$

$$f_6 = \frac{\eta^{18}(4z)\eta^{14}(6z)\eta^{2}(12z)}{\eta^{6}(2z)},$$

$$f_7 = \frac{\eta(4z)\eta^{19}(6z)\eta^{13}(12z)}{\eta^{5}(2z)},$$

$$f_8 = \frac{\eta^{13}(2z)\eta^{7}(4z)\eta^{13}(6z)}{\eta^{5}(12z)},$$

$$f_9 = \frac{\eta^{19}(4z)\eta^{13}(6z)\eta^{7}(12z)}{\eta^{11}(2z)},$$

$$f_{10} = \frac{\eta^{20}(4z)\eta^{12}(6z)\eta^{12}(12z)}{\eta^{16}(2z)},$$

$$f_{11} = \frac{\eta^{8}(2z)\eta^{20}(4z)\eta^{12}(12z)}{\eta^{12}(6z)},$$

$$f_{12} = \frac{\eta^{12}(2z)\eta^{12}(4z)\eta^{20}(12z)}{\eta^{16}(6z)},$$

$$f_{13} = \frac{\eta^{18}(4z)\eta^{20}(6z)\eta^{2}(12z)}{\eta^{12}(2z)},$$

$$f_{14} = \frac{\eta^{20}(4z)\eta^{6}(6z)\eta^{12}(12z)}{\eta^{10}(2z)},$$

$$f_{15} = \frac{\eta^{15}(4z)\eta^{11}(6z)\eta^{11}(12z)}{\eta^{9}(2z)},$$

$$f_{16} = \frac{\eta^{10}(4z)\eta^{16}(6z)\eta^{10}(12z)}{\eta^{8}(2z)},$$

$$f_{17} = \frac{\eta^{11}(2z)\eta^{15}(6z)\eta^{15}(12z)}{\eta^{13}(4z)},$$

$$f_{18} = \frac{\eta^{10}(2z)\eta^{16}(4z)\eta^{16}(12z)}{\eta^{14}(6z)},$$

$$f_{19} = \frac{\eta^{2}(4z)\eta^{12}(6z)\eta^{18}(12z)}{\eta^{4}(2z)},$$

$$f_{20} = \frac{\eta^{17}(6z)\eta^{17}(12z)}{\eta^{3}(2z)\eta^{3}(4z)},$$

$$f_{21} = \frac{\eta^{5}(2z)\eta^{7}(4z)\eta^{19}(6z)}{\eta^{5}(12z)},$$

$$f_{22} = \frac{\eta^{16}(4z)\eta^{16}(12z)}{\eta^{2}(2z)\eta^{2}(6z)},$$

$$f_{23} = \eta^{18}(4z)\eta^{8}(6z)\eta^{2}(12z).$$

Now we can state our main Theorem:

Theorem 1. Let $b_1, b_2, \cdots, b_5$ be non-negative integers satisfying

$$b_1 + b_2 + \cdots + b_5 \le 28. \tag{5}$$

Define the integers $a_1, a_2, a_3, a_4, a_6, a_{12}$ by

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 28, \tag{6}$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 70, \tag{7}$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 84, \tag{8}$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 28, \tag{9}$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 210, \tag{10}$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 84. \tag{11}$$

The functions defined before are functions of *q* by (3). Now define integers

$k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18}, k_{19}, k_{20}, k_{21}, k_{22}, k_{23}, k_{24}, k_{25}, k_{26}, k_{27}$ and k_{28}

by

$$\frac{1}{2^{b_1+b_5}} x^{b_1}(1-x)^{b_2}(1+x)^{b_3}(1+2x)^{b_4}(2+x)^{b_5} \tag{12}$$

$$= k_0 + k_1 x + k_2 x^2 + k_3 x + k_4 x^4 + k_5 x^5 + k_6 x^6 + k_7 x^7 + k_8 x^8 + k_9 x^9 + k_{10} x^{10} + k_{11} x^{11} + k_{12} x^{12} + k_{13} x^{13} + k_{14} x^{14} + k_{15} x^{15} \tag{13}$$

$$+ k_{16} x^{16} + k_{17} x^{17} + k_{18} x^{18} + k_{19} x^{19} + k_{20} x^{20} \tag{14}$$

$$+ k_{21} x^{21} + k_{22} x^{22} + k_{23} x^{23} + k_{24} x^{24} + k_{25} x^{25} + k_{26} x^{26} + k_{27} x^{27} + k_{28} x^{28}. \tag{15}$$

Define the rational numbers

$c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, \ldots, r_{22}$

and r_{23} as in Table **1**. Here $\{f_1, \ldots f_{23}\} \setminus \{f_{10}, f_{11}, f_{12}, f_{17}, f_{18}\} \in S_{14}(\Gamma_0(12)), f_{10}, f_{11}, f_{12}, f_{17}, f_{18} \in M_{14}(\Gamma_0(12)) \setminus S_{14}(\Gamma_0(12))$ and

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n,$$

where for $n \in \mathbb{N}$,

$$c(n) = -c_1\sigma_{13}(n) - c_2\sigma_{13}\left(\frac{n}{2}\right) - c_3\sigma_{13}\left(\frac{n}{3}\right) - c_4\sigma_{13}\left(\frac{n}{4}\right) - c_6\sigma_{13}\left(\frac{n}{6}\right) - c_{12}\sigma_{13}\left(\frac{n}{12}\right) + r_1f_1(n) + \cdots + r_{23}f_{23}(n).$$

In particular,

$$c(2n) = -c_1\sigma_{13}(2n) - c_2\sigma_{13}(n) - c_4\sigma_{13}\left(\frac{n}{2}\right) - (16385c_3 + c_6)\sigma_{13}\left(\frac{n}{3}\right) - (c_{12} - 16384c_3)\sigma_{13}\left(\frac{n}{6}\right) + r_1f_1(2n) + \cdots + r_{12}f_{12}(2n),$$

$$c(2n-1) = -c_1\sigma_{13}(2n-1) - c_3\sigma_{13}\left(\frac{2n-1}{3}\right) + r_{13}f_{13}(2n-1) + \cdots + r_{23}f_{23}(2n-1),$$

for $n \in \mathbb{N}$.

Proof. It follows from (6-11) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1, \quad (16)$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 28,$$
$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - 2\frac{a_4}{3} - \frac{a_6}{3} - 2\frac{a_{12}}{3} = -b_1 - b_5. \quad (17)$$

Now we will use p-k parametrization of Alaca, Alaca and Williams, see [15]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, \qquad k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \quad (18)$$

where the theta function $\varphi(q)$ is defined by

$$\varphi(q) = \sum_{-\infty}^{\infty} q^{n^2}.$$

Setting x=p in (12), and multiplying both sides by k^{14}, we obtain

$$\frac{k^{14}}{2^{b_1+b_5}} p^{b_1}(1-p)^{b_2}(1+p)^{b_3}(1+2p)^{b_4}(2+p)^{b_5} = (k_0 + k_1p + k_2p^2 + k_3p^3 + k_4p^4 + k_5p^5 + k_6p^6 + k_7p^7 + k_8p^8 + k_9p^9 + k_{10}p^{10} + k_{11}p^{11} + k_{12}p^{12} + k_{13}p^{13} + k_{14}p^{14} + k_{15}p^{15} + k_{16}p^{16} + k_{17}p^{17} + k_{18}p^{18} + k_{19}p^{19} + k_{20}p^{20} + k_{21}p^{21} + k_{22}p^{22} + k_{23}p^{23} + k_{24}p^{24} + k_{25}p^{25} + k_{26}p^{26} + k_{27}p^{27} + k_{28}p^{28})k^{14}.$$

Alaca, Alaca and Williams [16] have established the following representations in terms of p and k:

$$\eta(q) = 2^{-1/6}p^{1/24}(1-p)^{1/2}(1+p)^{1/6}(1+2p)^{1/8}(2+p)^{1/8}k^{1/2}, \quad (19)$$

$$\eta(q^2) = 2^{-1/3}p^{1/12}(1-p)^{1/4}(1+p)^{1/12}(1+2p)^{1/4}(2+p)^{1/4}k^{1/2}, \quad (20)$$

$$\eta(q^3) = 2^{-1/6}p^{1/8}(1-p)^{1/6}(1+p)^{1/2}(1+2p)^{1/24}(2+p)^{1/24}k^{1/2}, \quad (21)$$

$$\eta(q^4) = 2^{-2/3}p^{1/6}(1-p)^{1/8}(1+p)^{1/24}(1+2p)^{1/8}(2+p)^{1/2}k^{1/2}, \quad (22)$$

$$\eta(q^6) = 2^{-1/3}p^{1/4}(1-p)^{1/12}(1+p)^{1/4}(1+2p)^{1/12}(2+p)^{1/12}k^{1/2}, \quad (23)$$

$$\eta(q^{12}) = 2^{-2/3}p^{1/2}(1-p)^{1/24}(1+p)^{1/8}(1+2p)^{1/24}(2+p)^{1/6}k^{1/2}, \quad (24)$$

$$E_6(q) := 1 - 504\sum_{n=1}^{\infty}\sigma_5(n)q^n$$
$$= (1 - 246p - 5532p^2 - 38614p^3 - 135369p^4 - 276084p^5 - 348024p^6 - 276084p^7 - 135369p^8 - 38614p^9 - 5532p^{10} - 246p^{11} + p^{12})k^6,$$

$$E_4(q) := 1 + 240\sum_{n=1}^{\infty}\sigma_3(n)q^n$$
$$= (1 + 124p + 964p^2 + 2788p^3 + 3910p^4 + 2788p^5 + 964p^6 + 124p^7 + p^8)k^4.$$

Therefore, since

$$E_{14}(q) = E_6(q)E_4^2(q),$$

we immediately obtain:

$$E_{14}(q) = (p^{28} + 2p^{27} - 49236p^{26} - 5422686p^{25} - 163992237p^{24} - 2449687308p^{23} - 22413386328p^{22} - 139906977036p^{21} - 634557236991p^{20} - 2176932094146p^{19} - 5802918047148p^{18} - 12242753380770p^{17} -$$

$20701138105941p^{16} - 28283559161640p^{15} - 31368831795024p^{14} - 28283559161640p^{13} - 20701138105941p^{12} - 12242753380770p^{11} - 5802918047148p^{10} - 2176932094146p^{9} - 634557236991p^{8} - 139906977036p^{7} - 22413386328p^{6} - 2449687308p^{5} - 163992237p^{4} - 5422686p^{3} - 49236p^{2} + 2p + 1)k^{14}$,

$$E_{14}(q^2) = (p^{28} + 14p^{27} + 78p^{26} + 195p^{25} - \frac{24495}{2}p^{24} - 148530p^{23} - 1344144p^{22} - 8539014p^{21} - \frac{77628543}{2}p^{20} - 132904545p^{19} - 354052566p^{18} - 747152337p^{17} - \frac{2527176021}{2}p^{16} - 1726344084p^{15} - 1914554280p^{14} - 1726344084p^{13} - \frac{2527176021}{2}p^{12} - 747152337p^{11} - 354052566p^{10} - 132904545p^{9} - \frac{77628543}{2}p^{8} - 8539014p^{7} - 1344144p^{6} - 148530p^{5} - \frac{24495}{2}p^{4} + 195p^{3} + 78p^{2} + 14p + 1)k^{14},$$

$$E_{14}(q^3) = (p^{28} + 14p^{27} + 84p^{26} + 270p^{25} + 435p^{24} + 12p^{23} - 4488p^{22} - 36468p^{21} - 155679p^{20} - 432942p^{19} - 1081044p^{18} - 2584878p^{17} - 4682133p^{16} - 5908056p^{15} - 6062064p^{14} - 5908056p^{13} - 4682133p^{12} - 2584878p^{11} - 1081044p^{10} - 432942p^{9} - 155679p^{8} - 36468p^{7} - 4488p^{6} + 12p^{5} + 435p^{4} + 270p^{3} + 84p^{2} + 14p + 1)k^{14},$$

$$E_{14}(q^4) = (\frac{1}{16384}p^{28} + \frac{13}{8192}p^{27} - \frac{3057}{1024}p^{26} + \frac{1036281}{4096}p^{25} - \frac{22206081}{8192}p^{24} + \frac{1591125}{2048}p^{23} + \frac{50484903}{2048}p^{22} + \frac{42783}{32}p^{21} - \frac{11056359}{128}p^{20} - 44838p^{19} + \frac{3658233}{32}p^{18} + \frac{454011}{8}p^{17} - \frac{727467}{4}p^{16} - \frac{914361}{4}p^{15} - \frac{745761}{8}p^{14} - \frac{278211}{8}p^{13} - \frac{4044255}{64}p^{12} - \frac{1981695}{32}p^{11} - \frac{224763}{8}p^{10} - \frac{104643}{16}p^{9} - \frac{74613}{32}p^{8} - \frac{17193}{8}p^{7} - \frac{7635}{8}p^{6} + 192p^{5} + 471p^{4} + 273p^{3} + 84p^{2} + 14p + 1)k^{14},$$

$$E_{14}(q^6) = (p^{28} + 14p^{27} + 84p^{26} + 273p^{25} + \frac{945}{2}p^{24} + 210p^{23} - 864p^{22} - 1914p^{21} - \frac{2559}{2}p^{20} + 1065p^{19} + 2646p^{18} + 1641p^{17} - \frac{1173}{2}p^{16} - 1836p^{15} - 2040p^{14} - 1836p^{13} - \frac{1173}{2}p^{12} + 1641p^{11} + 2646p^{10} + 1065p^{9} - \frac{2559}{2}p^{8} - 1914p^{7} - 864p^{6} + 210p^{5} + \frac{945}{2}p^{4} + 273p^{3} + 84p^{2} + 14p + 1)k^{14},$$

$$E_{14}(q^{12}) = (\frac{1}{16384}p^{28} + \frac{7}{8192}p^{27} + \frac{21}{4096}p^{26} + \frac{69}{4096}p^{25} + \frac{255}{8192}p^{24} + \frac{51}{2048}p^{23} - \frac{423}{2048}p^{22} - \frac{273}{128}p^{21} - \frac{4863}{512}p^{20} - \frac{2241}{128}p^{19} + \frac{2607}{128}p^{18} + \frac{1419}{8}p^{17} + \frac{5667}{16}p^{16} + 105p^{15} - \frac{6681}{8}p^{14} - \frac{12645}{8}p^{13} - \frac{41919}{64}p^{12} + \frac{51891}{32}p^{11} + \frac{43071}{16}p^{10} + \frac{17745}{16}p^{9} - \frac{40341}{32}p^{8} - \frac{15279}{8}p^{7} - \frac{6909}{8}p^{6} + 210p^{5} + \frac{945}{2}p^{4} + 273p^{3} + 84p^{2} + 14p + 1)k^{14}.$$

It is easy to check the following expressions by (19-24)

$$f_1 := \sum_{n=0}^{\infty} f_1(n) = \frac{\eta^{19}(4z)\eta^{13}(6z)\eta^{7}(12z)}{\eta^{11}(2z)} = (-\frac{1}{131072}p^{24} - \frac{43}{262144}p^{23} - \frac{421}{262144}p^{22} - \frac{619}{65536}p^{21} - \frac{2425}{65536}p^{20} - \frac{26443}{262144}p^{19} - \frac{50501}{262144}p^{18} - \frac{32463}{131072}p^{17} - \frac{2925}{16384}p^{16} + \frac{17}{1024}p^{15} + \frac{1709}{8192}p^{14} + \frac{1075}{4096}p^{13} + \frac{187}{1024}p^{12} + \frac{79}{1024}p^{11} + \frac{19}{1024}p^{10} + \frac{1}{512}p^{9})k^{14},$$

$$f_2 := \sum_{n=0}^{\infty} f_2(n) = \frac{\eta^{14}(4z)\eta^{18}(6z)\eta^{6}(12z)}{\eta^{10}(2z)} = (-\frac{1}{32768}p^{23} - \frac{37}{65536}p^{22} - \frac{77}{16384}p^{21} - \frac{1517}{65536}p^{20} - \frac{1219}{16384}p^{19} - \frac{10571}{65536}p^{18} - \frac{3789}{16384}p^{17} - \frac{12519}{65536}p^{16} - \frac{513}{32768}p^{15} + \frac{2933}{16384}p^{14} + \frac{2027}{8192}p^{13} + \frac{731}{4096}p^{12} + \frac{157}{2048}p^{11} + \frac{19}{1024}p^{10} + \frac{1}{512}p^{9})k^{14},$$

$$f_3 := \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{16}(4z)\eta^{4}(6z)\eta^{16}(12z)}{\eta^{8}(2z)} = (-\frac{1}{524288}p^{25} - \frac{41}{1048576}p^{24} - \frac{95}{262144}p^{23} - \frac{131}{65536}p^{22} - \frac{1901}{262144}p^{21} - \frac{18839}{1048576}p^{20} - \frac{15831}{524288}p^{19} - \frac{2079}{65536}p^{18} - \frac{423}{32768}p^{17} + \frac{559}{32768}p^{16} + \frac{575}{16384}p^{15} + \frac{125}{4096}p^{14} + \frac{31}{2048}p^{13} + \frac{17}{4096}p^{12} + \frac{1}{2048}p^{11})k^{14},$$

$$f_4 := \sum_{n=0}^{\infty} f_4(n) = \frac{\eta^{12}(2z)\eta^{12}(4z)\eta^{8}(6z)}{\eta^{4}(12z)} = (-\frac{1}{128}p^{25} - \frac{37}{256}p^{24} - \frac{297}{256}p^{23} - \frac{2641}{512}p^{22} - \frac{26597}{2048}p^{21} - \frac{56529}{4096}p^{20} + \frac{18423}{1024}p^{19} + \frac{166545}{2048}p^{18} + \frac{191259}{2048}p^{17} - \frac{9071}{256}p^{16} - \frac{427163}{2048}p^{15} - \frac{372501}{2048}p^{14} + \frac{70795}{1024}p^{13} + \frac{987377}{4096}p^{12} + \frac{281673}{2048}p^{11} - \frac{15219}{256}p^{10} - \frac{14679}{128}p^{9} - \frac{5985}{128}p^{8} + \frac{727}{64}p^{7} + \frac{299}{16}p^{6} + \frac{63}{8}p^{5} + \frac{25}{16}p^{4} + \frac{1}{8}p^{3})k^{14},$$

$$f_5 := \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^6(4z)\eta^{14}(6z)\eta^{14}(12z)}{\eta^6(2z)}$$
$$= (-\frac{1}{32768}p^{23} - \frac{29}{65536}p^{22} - \frac{23}{8192}p^{21} - \frac{665}{65536}p^{20} - \frac{185}{8192}p^{19} - \frac{1991}{65536}p^{18} - \frac{159}{8192}p^{17} + \frac{533}{65536}p^{16} + \frac{965}{32768}p^{15} + \frac{235}{8192}p^{14} + \frac{61}{4096}p^{13} + \frac{17}{4096}p^{12} + \frac{1}{2048}p^{11})k^{14},$$

$$f_6 := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{18}(4z)\eta^{14}(6z)\eta^{2}(12z)}{\eta^6(2z)}$$
$$= (\frac{1}{16384}p^{24} + \frac{21}{16384}p^{23} + \frac{797}{65536}p^{22} + \frac{561}{8192}p^{21} + \frac{16503}{65536}p^{20} + \frac{20355}{32768}p^{19} + \frac{64859}{65536}p^{18} + \frac{13227}{16384}p^{17} - \frac{21827}{65536}p^{16} - \frac{60215}{32768}p^{15} - \frac{9489}{4096}p^{14} - \frac{2455}{2048}p^{13} + \frac{867}{2048}p^{12} + \frac{1191}{1024}p^{11} + \frac{57}{64}p^{10} + \frac{47}{128}p^{9} + \frac{21}{256}p^{8} + \frac{1}{128}p^{7})k^{14},$$

$$f_7 := \sum_{n=0}^{\infty} f_7(n) = \frac{\eta(4z)\eta^{19}(6z)\eta^{13}(12z)}{\eta^5(2z)}$$
$$= (-\frac{1}{8192}p^{22} - \frac{23}{16384}p^{21} - \frac{113}{16384}p^{20} - \frac{305}{16384}p^{19} - \frac{473}{16384}p^{18} - \frac{359}{16384}p^{17} + \frac{65}{16384}p^{16} + \frac{437}{16384}p^{15} + \frac{455}{16384}p^{14} + \frac{121}{8192}p^{13} + \frac{17}{4096}p^{12} + \frac{1}{2048}p^{11})k^{14},$$

$$f_8 := \sum_{n=0}^{\infty} f_8(n) = \frac{\eta^{13}(2z)\eta^{7}(4z)\eta^{13}(6z)}{\eta^5(12z)}$$
$$= (-\frac{1}{32}p^{24} - \frac{31}{64}p^{23} - \frac{101}{32}p^{22} - \frac{1371}{128}p^{21} - \frac{8761}{512}p^{20} + \frac{4237}{1024}p^{19} + \frac{70303}{1024}p^{18} + \frac{54311}{512}p^{17} - \frac{23}{256}p^{16} - \frac{98369}{512}p^{15} - \frac{106071}{512}p^{14} + \frac{1131}{32}p^{13} + \frac{120989}{512}p^{12} + \frac{157877}{1024}p^{11} - \frac{48697}{1024}p^{10} - \frac{58439}{512}p^{9} - \frac{12733}{256}p^{8} + \frac{1249}{128}p^{7} + \frac{1173}{64}p^{6} + \frac{251}{32}p^{5} + \frac{25}{16}p^{4} + \frac{1}{8}p^{3})k^{14},$$

$$f_9 := \sum_{n=0}^{\infty} f_9(n) = \frac{\eta^{20}(4z)\eta^{12}(12z)}{\eta^4(2z)}$$
$$= (\frac{1}{262144}p^{26} + \frac{23}{262144}p^{25} + \frac{961}{1048576}p^{24} + \frac{2999}{524288}p^{23} + \frac{12343}{524288}p^{22} + \frac{8663}{131072}p^{21} + \frac{130417}{1048576}p^{20} + \frac{72523}{524288}p^{19} + \frac{7183}{262144}p^{18} - \frac{24827}{131072}p^{17} - \frac{11011}{32768}p^{16} - \frac{4129}{16384}p^{15} - \frac{101}{8192}p^{14} + \frac{637}{4096}p^{13} + \frac{629}{4096}p^{12} + \frac{151}{2048}p^{11} + \frac{19}{1024}p^{10} + \frac{1}{512}p^{9})k^{14},$$

$$f_{10} := \sum_{n=0}^{\infty} f_{10}(n) = \frac{\eta^{20}(4z)\eta^{12}(6z)\eta^{12}(12z)}{\eta^{16}(2z)}$$
$$= (\frac{1}{1048576}p^{24} + \frac{11}{524288}p^{23} + \frac{111}{524288}p^{22} + \frac{85}{65536}p^{21} + \frac{5641}{1048576}p^{20} + \frac{8361}{524288}p^{19} + \frac{2277}{65536}p^{18} + \frac{1845}{32768}p^{17} + \frac{2223}{32768}p^{16} + \frac{983}{16384}p^{15} + \frac{155}{4096}p^{14} + \frac{33}{2048}p^{13} + \frac{17}{4096}p^{12} + \frac{1}{2048}p^{11})k^{14},$$

$$f_{11} := \sum_{n=0}^{\infty} f_{11}(n) = \frac{\eta^{8}(2z)\eta^{20}(4z)\eta^{12}(12z)}{\eta^{12}(6z)}$$
$$= (\frac{1}{65536}p^{28} + \frac{3}{8192}p^{27} + \frac{519}{131072}p^{26} + \frac{3307}{131072}p^{25} + \frac{108369}{1048576}p^{24} + \frac{143523}{524288}p^{23} + \frac{219543}{524288}p^{22} + \frac{3879}{32768}p^{21} - \frac{1023255}{1048576}p^{20} - \frac{1120483}{524288}p^{19} - \frac{200733}{131072}p^{18} + \frac{88959}{65536}p^{17} + \frac{245843}{65536}p^{16} + \frac{83799}{32768}p^{15} - \frac{3897}{4096}p^{14} - \frac{5523}{2048}p^{13} - \frac{6273}{4096}p^{12} + \frac{315}{2048}p^{11} + \frac{323}{512}p^{10} + \frac{87}{256}p^{9} + \frac{21}{256}p^{8} + \frac{1}{128}p^{7})k^{14},$$

$$f_{12} := \sum_{n=0}^{\infty} f_{12}(n) = \frac{\eta^{12}(2z)\eta^{12}(4z)\eta^{20}(12z)}{\eta^{16}(6z)}$$
$$= (\frac{1}{65536}p^{28} + \frac{5}{16384}p^{27} + \frac{351}{131072}p^{26} + \frac{1743}{131072}p^{25} + \frac{41361}{1048576}p^{24} + \frac{32913}{523288}p^{23} + \frac{5169}{524288}p^{22} - \frac{11283}{65536}p^{21} - \frac{342495}{1048576}p^{20} - \frac{74437}{524288}p^{19} + \frac{89903}{262144}p^{18} + \frac{72549}{131072}p^{17} + \frac{5421}{32768}p^{16} - \frac{5217}{16384}p^{15} - \frac{2781}{8192}p^{14} - \frac{267}{4096}p^{13} + \frac{357}{4096}p^{12} + \frac{135}{2048}p^{11} + \frac{19}{1024}p^{10} + \frac{1}{512}p^{9})k^{14},$$

$$f_{13} := \sum_{n=0}^{\infty} f_{13}(n) = \frac{\eta^{18}(4z)\eta^{20}(6z)\eta^{2}(12z)}{\eta^{12}(2z)}$$
$$= (-\frac{1}{32768}p^{23} - \frac{41}{65536}p^{22} - \frac{191}{32768}p^{21} - \frac{2133}{65536}p^{20} - \frac{3955}{32768}p^{19} - \frac{20323}{65536}p^{18} - \frac{18149}{32768}p^{17} - \frac{42831}{65536}p^{16} - \frac{1629}{4096}p^{15} + \frac{605}{4096}p^{14} + \frac{155}{256}p^{13} + \frac{1379}{2048}p^{12} + \frac{111}{256}p^{11} + \frac{11}{64}p^{10} + \frac{5}{128}p^{9} + \frac{1}{256}p^{8})k^{14},$$

$$f_{14} := \sum_{n=0}^{\infty} f_{14}(n) = \frac{\eta^{20}(4z)\eta^{6}(6z)\eta^{12}(12z)}{\eta^{10}(2z)}$$
$$= (-\frac{1}{524288}p^{25} - \frac{45}{1048576}p^{24} - \frac{231}{524288}p^{23} - \frac{357}{131072}p^{22} - \frac{2949}{262144}p^{21} - \frac{34047}{1048576}p^{20} - \frac{17335}{262144}p^{19} - \frac{24147}{262144}p^{18} - \frac{1251}{16384}p^{17} - \frac{287}{32768}p^{16} + \frac{567}{8192}p^{15} + \frac{825}{8192}p^{14} + \frac{39}{512}p^{13} + \frac{131}{4096}p^{12} + \frac{9}{1024}p^{11} + \frac{1}{1024}p^{10})k^{14},$$

$$f_{15} := \sum_{n=0}^{\infty} f_{15}(n) = \frac{\eta^{15}(4z)\eta^{11}(6z)\eta^{11}(12z)}{\eta^{9}(2z)}$$
$$= (-\frac{1}{131072}p^{24} - \frac{39}{262144}p^{23} - \frac{343}{262144}p^{22} - \frac{895}{131072}p^{21}$$

$$-\frac{765}{32768}p^{20}-\frac{14203}{262144}p^{19}-\frac{22095}{262144}p^{18}-\frac{81}{1024}p^{17}-\frac{333}{16384}p^{16}+\frac{469}{8192}p^{15}+\frac{771}{8192}p^{14}+\frac{19}{256}p^{13}+\frac{35}{1024}p^{12}+\frac{9}{1024}p^{11}+\frac{1}{1024}p^{10})k^{14},$$

$$f_{16}:=\sum_{n=0}^{\infty}f_{16}(n)=\frac{\eta^{10}(4z)\eta^{16}(6z)\eta^{10}(12z)}{\eta^{8}(2z)}$$
$$=(-\frac{1}{32768}p^{23}-\frac{33}{65536}p^{22}-\frac{121}{32768}p^{21}-\frac{1033}{65536}p^{20}-\frac{1405}{32768}p^{19}-\frac{4951}{65536}p^{18}-\frac{2627}{32768}p^{17}-\frac{2011}{65536}p^{16}+\frac{749}{16384}p^{15}+\frac{1435}{16384}p^{14}+\frac{37}{512}p^{13}+\frac{139}{4096}p^{12}+\frac{9}{1024}p^{11}+\frac{1}{1024}p^{10})k^{14},$$

$$f_{17}:=\sum_{n=0}^{\infty}f_{17}(n)=\frac{\eta^{11}(2z)\eta^{15}(6z)\eta^{15}(12z)}{\eta^{13}(4z)}$$
$$=(-\frac{1}{128}p^{22}-\frac{9}{256}p^{21}-\frac{21}{512}p^{20}+\frac{45}{1024}p^{19}+\frac{141}{1024}p^{18}+\frac{9}{128}p^{17}-\frac{23}{256}p^{16}-\frac{63}{512}p^{15}-\frac{15}{512}p^{14}+\frac{9}{256}p^{13}+\frac{15}{512}p^{12}+\frac{9}{1024}p^{11}+\frac{1}{1024}p^{10})k^{14},$$

$$f_{18}:=\sum_{n=0}^{\infty}f_{18}(n)=\frac{\eta^{10}(2z)\eta^{16}(4z)\eta^{16}(12z)}{\eta^{14}(6z)}$$
$$=(\frac{1}{65536}p^{28}+\frac{11}{32768}p^{27}+\frac{431}{131072}p^{26}+\frac{2445}{131072}p^{25}+\frac{69249}{1048576}p^{24}+\frac{37137}{262144}p^{23}+\frac{70995}{524288}p^{22}-\frac{39963}{262144}p^{21}-\frac{703551}{1048576}p^{20}-\frac{104233}{131072}p^{19}+\frac{7733}{131072}p^{18}+\frac{40613}{32768}p^{17}+\frac{83391}{65536}p^{16}+\frac{51}{4096}p^{15}-\frac{3999}{4096}p^{14}-\frac{381}{512}p^{13}-\frac{177}{4096}p^{12}+\frac{123}{512}p^{11}+\frac{77}{512}p^{10}+\frac{5}{128}p^{9}+\frac{1}{256}p^{8})k^{14},$$

$$f_{19}:=\sum_{n=0}^{\infty}f_{19}(n)=\frac{\eta^{2}(4z)\eta^{12}(6z)\eta^{18}(12z)}{\eta^{4}(2z)}$$
$$=(-\frac{1}{32768}p^{23}-\frac{25}{65536}p^{22}-\frac{67}{32768}p^{21}-\frac{397}{65536}p^{20}-\frac{343}{32768}p^{19}-\frac{619}{65536}p^{18}-\frac{17}{32768}p^{17}+\frac{601}{65536}p^{16}+\frac{91}{8192}p^{15}+\frac{53}{8192}p^{14}+\frac{1}{512}p^{13}+\frac{1}{4096}p^{12})k^{14},$$

$$f_{20}:=\sum_{n=0}^{\infty}f_{11}(n)=\frac{\eta^{17}(6z)\eta^{17}(12z)}{\eta^{3}(2z)\eta^{3}(4z)}$$
$$=(-\frac{1}{8192}p^{22}-\frac{19}{16384}p^{21}-\frac{75}{16384}p^{20}-\frac{155}{16384}p^{19}-\frac{163}{16384}p^{18}-\frac{33}{16384}p^{17}+\frac{131}{16384}p^{16}+\frac{175}{16384}p^{15}+\frac{105}{16384}p^{14}+\frac{1}{512}p^{13}+\frac{1}{4096}p^{12})k^{14},$$

$$f_{21}:=\sum_{n=0}^{\infty}f_{21}(n)=\frac{\eta^{7}(2z)\eta^{7}(4z)\eta^{19}(6z)}{\eta^{5}(12z)}$$
$$=(\frac{1}{64}p^{23}+\frac{15}{64}p^{22}+\frac{191}{128}p^{21}+\frac{81}{16}p^{20}+\frac{9001}{1024}p^{19}+\frac{2445}{1024}p^{18}-\frac{5563}{256}p^{17}-\frac{5427}{128}p^{16}-\frac{10037}{512}p^{15}+\frac{20575}{512}p^{14}\frac{17297}{256}p^{13}+\frac{6461}{256}p^{12}-\frac{32919}{1024}p^{11}-\frac{42595}{1024}p^{10}-\frac{1779}{128}p^{9}+\frac{1831}{256}p^{8}+\frac{561}{64}p^{7}+\frac{235}{64}p^{6}+\frac{3}{4}p^{5}+\frac{1}{16}p^{4})k^{14},$$

$$f_{22}:=\sum_{n=0}^{\infty}f_{22}(n)=\frac{\eta^{16}(4z)\eta^{16}(12z)}{\eta^{2}(2z)\eta^{2}(6z)}$$
$$=(\frac{1}{262144}p^{26}+\frac{21}{262144}p^{25}+\frac{793}{1048576}p^{24}+\frac{1103}{262144}p^{23}+\frac{7931}{524288}p^{22}+\frac{9395}{262144}p^{21}+\frac{55257}{1048576}p^{20}+\frac{8633}{262144}p^{19}-\frac{10083}{262144}p^{18}-\frac{1843}{16384}p^{17}-\frac{3639}{32768}p^{16}-\frac{245}{8192}p^{15}+\frac{389}{8192}p^{14}+\frac{31}{512}p^{13}+\frac{133}{4096}p^{12}+\frac{9}{1024}p^{11}+\frac{1}{1024}p^{10})k^{14},$$

$$f_{23}:=\sum_{n=0}^{\infty}f_{23}(n)=\eta^{18}(4z)\eta^{8}(6z)\eta^{2}(12z)$$
$$=(-\frac{1}{8192}p^{25}-\frac{43}{16384}p^{24}-\frac{831}{32768}p^{23}-\frac{9445}{65536}p^{22}-\frac{17233}{32768}p^{21}-\frac{81405}{65536}p^{20}-\frac{55979}{32768}p^{19}-\frac{33299}{65536}p^{18}+\frac{97113}{32768}p^{17}+\frac{400557}{65536}p^{16}+\frac{76179}{16384}p^{15}-\frac{24267}{16384}p^{14}-\frac{1637}{256}p^{13}-\frac{11123}{2048}p^{12}-\frac{495}{512}p^{11}+\frac{971}{512}p^{10}+\frac{235}{128}p^{9}+\frac{201}{256}p^{8}+\frac{11}{64}p^{7}+\frac{1}{64}p^{6})k^{14}.$$

Obviously, $f_1,\dots f_{22}$ *and* f_{23} are functions of q, see (3), (18). We see that

$\{f_{10},f_{11},f_{12},f_{17},f_{18}\}\in S_{14}(\Gamma_0(12)), f_{10},\ f_{11},\ f_{12},\ f_{17},$ $f_{18}\in M_{14}(\Gamma_0(12))\backslash S_{14}(\Gamma_0(12))$ and $ord_{1/1}f_{10}=ord_{1/2}f_{10}$

$=ord_{1/3}f_{11}=ord_{1/6}f_{11}=ord_{1/3}f_{12}=ord_{1/6}f_{12}$

$=ord_{1/4}f_{17}\ =ord_{1/3}f_{18}=0$

by [17]. Now

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z)$$
$$=q^{b_1}\prod_{n=1}^{\infty}(1-q^{n})^{a_1}(1-q^{2n})^{a_2}(1-q^{3n})^{a_3}(1-q^{4n})$$
$$^{a_4}(1-q^{6n})^{a_6}(1-q^{12n})^{a_{12}}$$
$$=2^{-\frac{a_1}{6}-\frac{a_2}{3}-\frac{a_3}{6}-2\frac{a_4}{3}-\frac{a_6}{3}-2\frac{a_{12}}{3}}p^{24}{}^{\frac{a_1}{12}+\frac{a_2}{8}+\frac{a_3}{6}+\frac{a_4}{4}+\frac{a_6}{2}+\frac{a_{12}}{}}$$
$$(1-p)^{\frac{a_1}{2}+\frac{a_2}{4}+\frac{a_3}{6}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}}(1+p)^{\frac{a_1}{6}+\frac{a_2}{12}+\frac{a_3}{2}+\frac{a_4}{24}+\frac{a_6}{4}+\frac{a_{12}}{8}}$$

Table 2:

$$f_1 = \frac{233}{1020395520}\Delta_{2,14,1}(z) - \frac{233}{15943680}\Delta_{2,14,1}(2z) - \frac{136449}{113377280}\Delta_{2,14,1}(3z) + \frac{136449}{1771520}\Delta_{2,14,1}(6z) - \frac{179}{583925760}\Delta_{2,14,2}(z) - \frac{179}{9123840}\Delta_{2,14,2}(2z) - \frac{7773}{7208960}\Delta_{2,14,2}(3z) - \frac{7773}{112640}\Delta_{2,14,2}(6z) - \frac{61}{119439360}\Delta_{3,14,1}(z) - \frac{61}{9953280}\Delta_{3,14,1}(2z) - \frac{61}{14580}\Delta_{3,14,1}(4z) + \frac{1}{705528299520}(49t + 362724)\Delta_{3,14,2}(z) + \frac{1}{117588049920}(-60013t - 138768)\Delta_{3,14,2}(2z) + \frac{1}{86124060}(49t + 362724)\Delta_{3,14,2}(4z) + \frac{1}{705528299520}(-49t + 360078)\Delta_{3,14,3}(z) + \frac{1}{117588049920}(60013t + 3101934)\Delta_{3,14,3}(2z) + \frac{1}{86124060}(-49t + 360078)\Delta_{3,14,3}(4z) - \frac{29}{235634688}\Delta_{4,14}(z) + \frac{44277}{26181632}\Delta_{4,14}(3z) - \frac{25}{57397248}\Delta_{6,14}(z) + \frac{25}{896832}\Delta_{6,14}(2z) - \frac{215}{203046912}\Delta_{12,14,1}(z) + \frac{1327}{1122729984}\Delta_{12,14,2}(z),$$

$$f_2 = \frac{89}{446423040}\Delta_{2,14,1}(z) - \frac{89}{6975360}\Delta_{2,14,1}(2z) - \frac{1611}{1550080}\Delta_{2,14,1}(3z) + \frac{1611}{24220}\Delta_{2,14,1}(6z) - \frac{1}{3649536}\Delta_{2,14,2}(z) - \frac{1}{57024}\Delta_{2,14,2}(2z) - \frac{89}{90112}\Delta_{2,14,2}(3z) - \frac{89}{1408}\Delta_{2,14,2}(6z) - \frac{35}{76142592}\Delta_{3,14,1}(z) - \frac{35}{6345216}\Delta_{3,14,1}(2z) - \frac{140}{37179}\Delta_{3,14,1}(4z) + \frac{1}{7408047144960}(223t + 3389658)\Delta_{3,14,2}(z) + \frac{1}{154334315520}(-7036t - 78942)\Delta_{3,14,2}(2z) + \frac{1}{904302630}(223t + 3389658)\Delta_{3,14,2}(4z) + \frac{1}{7408047144960}(-223t + 3377616)\Delta_{3,14,3}(z) + \frac{1}{154334315520}(70367t + 3720876)\Delta_{3,14,3}(2z) + \frac{1}{904302630}(-223t + 3377616)\Delta_{3,14,3}(4z) - \frac{7}{58908672}\Delta_{4,14}(z) + \frac{9861}{6545408}\Delta_{4,14}(3z) - \frac{185}{487876608}\Delta_{6,14}(z) + \frac{185}{7623072}\Delta_{6,14}(2z) - \frac{95}{101523456}\Delta_{12,14,1}(z) + \frac{37}{35085312}\Delta_{12,14,2}(z),$$

$$f_3 = \frac{37}{1190461440}\Delta_{2,14,1}(z) - \frac{37}{18600960}\Delta_{2,14,1}(2z) - \frac{150021}{793640960}\Delta_{2,14,1}(3z) + \frac{150021}{12400640}\Delta_{2,14,1}(6z) - \frac{1}{26542080}\Delta_{2,14,2}(z) - \frac{1}{414720}\Delta_{2,14,2}(2z) - \frac{69}{655360}\Delta_{2,14,2}(3z) - \frac{69}{10240}\Delta_{2,14,2}(6z) - \frac{53}{1015234560}\Delta_{3,14,1}(z)$$

(Table 2). Continued.

$$-\frac{53}{84602880}\Delta_{3,14,1}(2z)-\frac{53}{123930}\Delta_{3,14,1}(4z)+\frac{1}{1234674524160}(74t+78789)\Delta_{3,14,2}(z)$$
$$+\frac{1}{411558174720}(-24931t-419136)\Delta_{3,14,2}(2z)+\frac{1}{150717105}(74t+78789)\Delta_{3,14,2}(4z)$$
$$+\frac{1}{1234674524160}(-74t+74793)\Delta_{3,14,3}(z)+\frac{1}{411558174720}(24931t+927138)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{150717105}(-74t+74793)\Delta_{3,14,3}(4z)-\frac{1}{353452032}\Delta_{4,14}(z)+\frac{5619}{26181632}\Delta_{4,14}(3z),$$
$$-\frac{1}{15246144}\Delta_{6,14}(z)+\frac{1}{238221}\Delta_{6,14}(2z)-\frac{7}{50761728}\Delta_{12,14,1}(z)+\frac{79}{561364992}\Delta_{12,14,2}(z),$$

$$f_4=\frac{683}{9300480}\Delta_{2,14,1}(z)-\frac{683}{145320}\Delta_{2,14,1}(2z)+\frac{1121931}{6200320}\Delta_{2,14,1}(3z)-\frac{1121931}{96880}\Delta_{2,14,1}(6z)$$
$$-\frac{19}{253440}\Delta_{2,14,2}(z)-\frac{19}{3960}\Delta_{2,14,2}(2z)+\frac{6561}{56320}\Delta_{2,14,2}(3z)+\frac{65651}{880}\Delta_{2,14,2}(6z)$$
$$+\frac{1}{48960}\Delta_{3,14,1}(z)+\frac{1}{4080}\Delta_{3,14,1}(2z)+\frac{128}{765}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{952680960}(-257t+25818)\Delta_{3,14,2}(z)+\frac{1}{19847520}(-827t+90978)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{1860705}(-4112t+413088)\Delta_{3,14,2}(4z)+\frac{1}{952680960}(257t+39696)\Delta_{3,14,3}(z)$$
$$+\frac{1}{19847520}(827t+135636)\Delta_{3,14,3}(2z)+\frac{1}{1860705}(4112t+635136)\Delta_{3,14,3}(4z)$$
$$-\frac{13}{613632}\Delta_{4,14}(z)+\frac{59049}{204544}\Delta_{4,14}(3z)-\frac{33}{376448}\Delta_{6,14}(z)+\frac{33}{5882}\Delta_{6,14}(2z)$$
$$+\frac{1}{13056}\Delta_{12,14,1}(z)-\frac{1}{18048}\Delta_{12,14,2}(z),$$

$$f_5=\frac{1}{37201920}\Delta_{2,14,1}(z)-\frac{1}{581280}\Delta_{2,14,1}(2z)-\frac{7291}{49602560}\Delta_{2,14,1}(3z)+\frac{7291}{775040}\Delta_{2,14,1}(6z)$$
$$-\frac{1}{36495360}\Delta_{2,14,2}(z)-\frac{1}{570240}\Delta_{2,14,2}(2z)-\frac{53}{675840}\Delta_{2,14,2}(3z)-\frac{53}{10560}\Delta_{2,14,2}(6z)$$
$$-\frac{19}{380712960}\Delta_{3,14,1}(z)-\frac{19}{31726080}\Delta_{3,14,1}(2z)-\frac{76}{185895}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{7408047144960}(139t+365154)\Delta_{3,14,2}(z)+\frac{1}{154334315520}(-7451t-49206)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{904302630}(139t+365154)\Delta_{3,14,2}(4z)+\frac{1}{7408047144960}(-139t+357648)\Delta_{3,14,3}(z)$$
$$+\frac{1}{154334315520}(7451t+353148)\Delta_{3,14,3}(2z)+\frac{1}{904302630}(-139t+357648)\Delta_{3,14,3}(4z)$$
$$-\frac{1}{176726016}\Delta_{4,14}(z)+\frac{3235}{19636224}\Delta_{4,14}(3z)-\frac{23}{487876608}\Delta_{6,14}(z)+\frac{23}{7623072}\Delta_{6,14}(2z)$$
$$-\frac{11}{101523456}\Delta_{12,14,1}(z)+\frac{1}{8771328}\Delta_{12,14,2}(z),$$

(Table 2). Continued.

$$f_6 = -\frac{7}{42516480}\Delta_{2,14,1}(z) + \frac{7}{664320}\Delta_{2,14,1}(2z) + \frac{22923}{14172160}\Delta_{2,14,1}(3z) - \frac{22923}{221440}\Delta_{2,14,1}(6z)$$
$$-\frac{17}{24330240}\Delta_{2,14,2}(z) - \frac{17}{380160}\Delta_{2,14,2}(2z) - \frac{2187}{901120}\Delta_{2,14,2}(3z) - \frac{2187}{14080}\Delta_{2,14,2}(6z)$$
$$+\frac{1}{9400320}\Delta_{3,14,1}(z) + \frac{1}{783360}\Delta_{3,14,1}(2z) + \frac{2}{2295}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{13065338880}(-71t - 306)\Delta_{3,14,2}(z) + \frac{1}{181463040}(-49t + 16756)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{1594890}(-71t - 306)\Delta_{3,14,2}(4z) + \frac{1}{13065338880}(71t + 3528)\Delta_{3,14,3}(z)$$
$$+\frac{1}{181463040}(49t + 19402)\Delta_{3,14,3}(2z) + \frac{1}{1594890}(71t + 3528)\Delta_{3,14,3}(4z)$$
$$-\frac{1}{6545408}\Delta_{4,14}(z) + \frac{351}{6545408}\Delta_{4,14}(3z) + \frac{83}{162625536}\Delta_{6,14}(z) - \frac{83}{2541024}\Delta_{6,14}(2z)$$
$$+\frac{1}{11280384}\Delta_{12,14,1}(z) + \frac{1}{15593472}\Delta_{12,14,2}(z),$$

$$f_7 = -\frac{11}{573972480}\Delta_{2,14,1}(z) - \frac{11}{8968320}\Delta_{2,14,1}(2z) - \frac{25889}{191324160}\Delta_{2,14,1}(3z)$$
$$+\frac{25889}{2989440}\Delta_{2,14,1}(6z) - \frac{1}{36495360}\Delta_{2,14,2}(z) - \frac{1}{570240}\Delta_{2,14,2}(2z)$$
$$-\frac{263}{4055040}\Delta_{2,14,2}(3z) - \frac{263}{63360}\Delta_{2,14,2}(6z) - \frac{13}{285534720}\Delta_{3,14,1}(z)$$
$$-\frac{13}{23794560}\Delta_{3,14,1}(2z) - \frac{208}{557685}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{1793719336960}(t + 36006)\Delta_{3,14,2}(z) + \frac{1}{16535819520}(-749t - 354)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{193779135}(2t + 72012)\Delta_{3,14,2}(4z) + \frac{1}{793719336960}(-t + 35952)\Delta_{3,14,3}(z)$$
$$+\frac{1}{16535819520}(749t + 40092)\Delta_{3,14,3}(2z) + \frac{1}{193779135}(-2t + 71904)\Delta_{3,14,3}(4z)$$
$$-\frac{5}{397633536}\Delta_{4,14}(z) + \frac{6587}{44181504}\Delta_{4,14}(3z) - \frac{1}{27104256}\Delta_{6,14}(z)$$
$$+\frac{1}{423504}\Delta_{6,14}(2z) - \frac{7}{76142592}\Delta_{12,14,1}(z) + \frac{11}{105255936}\Delta_{12,14,2}(z),$$

$$f_8 = -\frac{4931}{55802880}\Delta_{2,14,1}(z) - \frac{4931}{871920}\Delta_{2,14,1}(2z) + \frac{1030077}{6200320}\Delta_{2,14,1}(3z) - \frac{1030077}{96880}\Delta_{2,14,1}(6z)$$
$$-\frac{11}{138240}\Delta_{2,14,2}(z) - \frac{11}{2160}\Delta_{2,14,2}(2z) + \frac{729}{5120}\Delta_{2,14,2}(3z) + \frac{729}{80}\Delta_{2,14,2}(6z)$$
$$+\frac{1}{73440}\Delta_{3,14,1}(z) + \frac{1}{6120}\Delta_{3,14,1}(2z) + \frac{256}{2295}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{178627680}(-67t + 4098)\Delta_{3,14,2}(z) + \frac{1}{14885640}(-643t + 94872)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{5582115}(-17152t + 1049088)\Delta_{3,14,2}(4z) + \frac{1}{178627680}(67t + 7716)\Delta_{3,14,3}(z)$$
$$+\frac{1}{14885640}(643t + 129594)\Delta_{3,14,3}(2z) + \frac{1}{5582115}(17152t + 1975296)\Delta_{3,14,3}(4z)$$
$$-\frac{13}{690336}\Delta_{4,14}(z) + \frac{6561}{25568}\Delta_{4,14}(3z) - \frac{25}{282336}\Delta_{6,14}(z) + \frac{50}{8823}\Delta_{6,14}(2z)$$
$$+\frac{1}{14688}\Delta_{12,14,1}(z) - \frac{1}{20304}\Delta_{12,14,2}(z),$$

(Table 2). Continued.

$$f_9 = \frac{43}{1428553728}\Delta_{2,14,1}(z) - \frac{43}{22321152}\Delta_{2,14,1}(2z) + \frac{6561}{19841024}\Delta_{2,14,1}(3z) - \frac{6561}{310016}\Delta_{2,14,1}(6z) - \frac{1}{21626880}\Delta_{2,14,2}(z) - \frac{1}{337920}\Delta_{2,14,2}(2z) - \frac{243}{901120}\Delta_{2,14,2}(3z) - \frac{243}{14080}\Delta_{2,14,2}(6z) - \frac{7}{50135040}\Delta_{3,14,1}(z) - \frac{7}{4177920}\Delta_{3,14,1}(2z) - \frac{7}{6120}\Delta_{3,14,1}(4z) + \frac{1}{16259088384}(-5t+44)\Delta_{3,14,2}(z) + \frac{1}{8129544192}(-157t+42480)\Delta_{3,14,2}(2z) + \frac{1}{1984752}(-5t+44)\Delta_{3,14,2}(4z) + \frac{1}{16259088384}(5t+314)\Delta_{3,14,3}(z) + \frac{1}{8129544192}(157t+50958)\Delta_{3,14,3}(2z) + \frac{1}{1984752}(5t+314)\Delta_{3,14,3}(4z) - \frac{83}{706904064}\Delta_{4,14}(z) - \frac{729}{6545408}\Delta_{4,14}(3z) + \frac{29}{216834048}\Delta_{6,14}(z) - \frac{29}{3388032}\Delta_{6,14}(2z) + \frac{1}{30081024}\Delta_{12,14,1}(z) + \frac{7}{83165184}\Delta_{12,14,2}(z),$$

$$f_{10} = -\frac{457}{12599050240}\Delta_{2,14,1}(z) + \frac{457}{196860160}\Delta_{2,14,1}(2z) + \frac{18883857}{100792401920}\Delta_{2,14,1}(3z) - \frac{18883857}{1574881280}\Delta_{2,14,1}(6z) + \frac{89}{1569300480}\Delta_{2,14,2}(z) + \frac{89}{24520320}\Delta_{2,14,2}(2z) + \frac{61663}{309985280}\Delta_{2,14,2}(3z) + \frac{61663}{4843520}\Delta_{2,14,2}(6z) + \frac{563033}{6663999651840}\Delta_{3,14,1}(z) + \frac{563033}{555333304320}\Delta_{3,14,1}(2z) + \frac{563033}{813476520}\Delta_{3,14,1}(4z) + \frac{1}{64775964235530240}(2024959t-5551099716)\Delta_{3,14,2}(z) + \frac{1}{10795994039255040}(943407917t-5734683888)\Delta_{3,14,2}(2z) + \frac{1}{7907222196720}(2024959t-5551099716)\Delta_{3,14,2}(4z) + \frac{1}{64775964235530240}(-2024959t-5660447502)\Delta_{3,14,3}(z) + \frac{1}{10795994039255040}(-943407917t-56678711406)\Delta_{3,14,3}(2z) + \frac{1}{7907222196720}(-2024959t-5660447502)\Delta_{3,14,3}(4z) + \frac{5}{235634688}\Delta_{4,14}(z) - \frac{3693}{13090816}\Delta_{4,14}(3z) + \frac{133}{1951506432}\Delta_{6,14}(z) - \frac{133}{30492288}\Delta_{6,14}(2z) + \frac{143}{812187648}\Delta_{12,14,1}(z) - \frac{443}{2245459968}\Delta_{12,14,2}(z) - \frac{3}{855893221310464}E_{14}(z) + \frac{24579}{855893221310464}E_{14}(2z) + \frac{3}{855893221310464}E_{14}(3z) - \frac{3}{104479152992}E_{14}(4z) - \frac{24579}{855893221310464}E_{14}(6z) + \frac{3}{104479152992}E_{14}(12z),$$

$$f_{11} = \frac{5905559}{151188602880}\Delta_{2,14,1}(z) - \frac{5905559}{2362321920}\Delta_{2,14,1}(2z) - \frac{3495287457}{100792401920}\Delta_{2,14,1}(3z) + \frac{3495287457}{1574881280}\Delta_{2,14,1}(6z) + \frac{14957}{464977920}\Delta_{2,14,2}(z) + \frac{14957}{7265280}\Delta_{2,14,2}(2z) + \frac{14703201}{309985280}\Delta_{2,14,2}(3z) + \frac{14703201}{4843520}\Delta_{2,14,2}(6z) + \frac{100363}{304096320}\Delta_{3,14,1}(z) + \frac{301089}{761774080}\Delta_{3,14,1}(2z) + \frac{100363}{371960}\Delta_{3,14,1}(4z)$$

(Table 2). Continued.

$$+\frac{1}{29618639339520}(58213t+10238162)\Delta_{3,14,2}(z)$$
$$+\frac{1}{4936439889920}(-170112121t-1648592)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{3615556560}(58213t+10238162)\Delta_{3,14,2}(4z)$$
$$+\frac{1}{29618639339520}(-58213t+10206727)\Delta_{3,14,3}(z)$$
$$+\frac{1}{4936439889920}(170112121t+90211953)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{3615556560}(-58213t+10206727)\Delta_{3,14,3}(4z)-\frac{4073}{78544896}\Delta_{4,14}(z)$$
$$+\frac{177147}{13090816}\Delta_{4,14}(3z)+\frac{729}{24092672}\Delta_{6,14}(z)-\frac{729}{376448}\Delta_{6,14}(2z)$$
$$-\frac{81}{1114112}\Delta_{12,14,1}(z)-\frac{243}{3080192}\Delta_{12,14,2}(z)+\frac{1}{2567679663931392}E_{14}(z)$$
$$-\frac{2731}{855893221310464}E_{14}(2z)-\frac{1594323}{855893221310464}E_{14}(3z)$$
$$+\frac{1}{313437458976}E_{14}(4z)+\frac{13062288339}{855893221310464}E_{14}(6z)$$
$$-\frac{1594323}{104479152992}E_{14}(12z),$$

$$f_{12}=\frac{6294619}{302377205760}\Delta_{2,14,1}(z)-\frac{6294619}{4724643840}\Delta_{2,14,1}(2z)$$
$$-\frac{242868537}{12599050240}\Delta_{2,14,1}(3z)$$
$$+\frac{242868537}{196860160}\Delta_{2,14,1}(6z)+\frac{61663}{2789867520}\Delta_{2,14,2}(z)+\frac{61663}{43591680}\Delta_{2,14,2}(2z)$$
$$+\frac{583929}{19374080}\Delta_{2,14,2}(3z)$$
$$+\frac{583929}{302720}\Delta_{2,14,2}(6z)+\frac{563033}{27423866880}\Delta_{3,14,1}(z)+\frac{563033}{2285322240}\Delta_{3,14,1}(2z)$$
$$+\frac{563033}{3347640}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{266567754055680}(-2024959t+5551099716)\Delta_{3,14,2}(z)$$
$$+\frac{1}{44427959009280}(-943407917t+5734683888)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{32540009040}(-2024959t+5551099716)\Delta_{3,14,2}(4z)$$
$$+\frac{1}{266567754055680}(2024959t+5660447502)\Delta_{3,14,3}(z)$$
$$+\frac{1}{44427959009280}(943407917t+56678711406)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{32540009040}(2024959t+5660447502)\Delta_{3,14,3}(4z)-\frac{1231}{39272448}\Delta_{4,14}(z)$$
$$+\frac{295245}{26181632}\Delta_{4,14}(3z)+\frac{399}{24092672}\Delta_{6,14}(z)-\frac{399}{376448}\Delta_{6,14}(2z)$$
$$-\frac{143}{3342336}\Delta_{12,14,1}(z)-\frac{443}{9240576}\Delta_{12,14,2}(z)+\frac{1}{2567679663931392}E_{14}(z)$$
$$-\frac{2731}{8558893221310464}E_{14}(2z)-\frac{1594323}{855893221310464}E_{14}(3z)$$
$$+\frac{1}{313437458976}E_{14}(4z)+\frac{13062288339}{855893221310464}E_{14}(6z)$$
$$-\frac{1594323}{104479152992}E_{14}(12z),$$

$$f_{13}=-\frac{33}{310016}\Delta_{2,14,1}(2z)+\frac{161379}{310016}\Delta_{2,14,1}(6z)-\frac{73}{380160}\Delta_{2,14,2}(2z)$$
$$-\frac{9663}{14080}\Delta_{2,14,2}(6z)$$

(Table 2). Continued.

$$-\frac{1}{123930}\Delta_{3,14,1}(2z)-\frac{2044}{61965}\Delta_{3,14,1}(4z)+\frac{1}{241147368}(-347t+4056)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{60286842}(-1825t+2028378)\Delta_{3,14,2}(4z)+\frac{1}{241147368}(347t+22794)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{60286842}(1825t+2126928)\Delta_{3,14,3}(4z)+\frac{31}{158814}\Delta_{6,14}(2z),$$

$$f_{14}=-\frac{31}{2657280}\Delta_{2,14,1}(2z)+\frac{61479}{885760}\Delta_{2,14,1}(6z)-\frac{47}{1520640}\Delta_{2,14,2}(2z)-\frac{5697}{56320}\Delta_{2,14,2}(6z)$$
$$+\frac{1}{220320}\Delta_{3,14,1}(2z)-\frac{253}{55080}\Delta_{3,14,1}(4z)+\frac{1}{76554720}(-17t+78)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{38277360}(-257t+186918)\Delta_{3,14,2}(4z)+\frac{1}{76554720}(17t+996)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{38277360}(257t+200796)\Delta_{3,14,3}(4z)+\frac{61}{2541024}\Delta_{6,14}(2z),$$

$$f_{15}=-\frac{69}{6200320}\Delta_{2,14,1}(2z)+\frac{390933}{6200320}\Delta_{2,14,1}(6z)-\frac{41}{1520640}\Delta_{2,14,2}(2z)-\frac{5061}{56320}\Delta_{2,14,2}(6z)$$
$$+\frac{7}{1982880}\Delta_{3,14,1}(2z)-\frac{1013}{247860}\Delta_{3,14,1}(4z)+\frac{1}{2411473680}(-463t+3072)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{602868420}(-3629t+2580036)\Delta_{3,14,2}(4z)+\frac{1}{2411473680}(463t+28074)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{602868420}(3629t+2776002)\Delta_{3,14,3}(4z)+\frac{55}{2541024}\Delta_{6,14}(2z),$$

$$f_{16}=-\frac{149}{13950720}\Delta_{2,14,1}(2z)+\frac{88007}{1550080}\Delta_{2,14,1}(6z)-\frac{1}{42240}\Delta_{2,14,2}(2z)-\frac{1119}{14080}\Delta_{2,14,2}(6z)$$
$$+\frac{1}{371790}\Delta_{3,14,1}(2z)-\frac{676}{185895}\Delta_{3,14,1}(4z)+\frac{1}{28937684160}(-4831t+43014)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{904302630}(-4931t+3401934)\Delta_{3,14,2}(4z)+\frac{1}{28937684160}(4831t+303888)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{904302630}(4931t+3668208)\Delta_{3,14,3}(4z)+\frac{25}{1270512}\Delta_{6,14}(2z),$$

$$f_{17}=\frac{16673}{110733840}\Delta_{2,14,1}(2z)-\frac{9504049}{12303760}\Delta_{2,14,1}(6z)-\frac{61}{204336}\Delta_{2,14,2}(2z)$$
$$-\frac{20629}{54231768}\Delta_{3,14,1}(2z)+\frac{15616}{20336913}\Delta_{3,14,1}(4z)-\frac{23347}{22704}\Delta_{2,14,2}(6z)$$
$$+\frac{1}{1317870366120}(-589433t+514791132)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{1494201387295}(-137324416t+1544427264)\Delta_{3,14,2}(4z)$$
$$+\frac{1}{1317870366120}(589433t+546620514)\Delta_{3,14,3}(2z)$$
$$+\frac{1}{494201387295}(1373244165t+8959945728)\Delta_{3,14,3}(4z)-\frac{22}{79407}\Delta_{6,14}(2z)$$
$$-\frac{1}{235078094232}E_{14}(2z)\,\frac{2048}{29384761779}E_{14}(4z)+\frac{1}{235078094232}E_{14}(6z)$$
$$-\frac{2048}{29384761779}E_{14}(12z),$$

(Table 2). Continued.

$$f_{18} = \frac{1413709}{295290240}\Delta_{2,14,1}(2z) - \frac{1696536819}{393720320}\Delta_{2,14,1}(6z) - \frac{391}{60544}\Delta_{2,14,2}(2z) - \frac{2184813}{242176}\Delta_{2,14,2}(6z) + \frac{137}{892704}\Delta_{3,14,1}(2z) - \frac{18551}{27897}\Delta_{3,14,1}(4z) + \frac{1}{21693339360}(653003t - 6366912)\Delta_{3,14,2}(4z) + \frac{1}{5423334840}(4258754t - 3730667721)\Delta_{3,14,3}(z) + \frac{1}{5423334840}(-4258754t - 3960640437)\Delta_{3,14,3}(4z)$$
$$\frac{351}{94112}\Delta_{6,14}(2z) - \frac{1}{313437458976}E_{14}(2z) + \frac{1}{313437458976}E_{14}(4z) + \frac{1594323}{104479152992}E_{14}(6z) - \frac{1594323}{104479152992}E_{14}(12z),$$

$$f_{19} = -\frac{1}{2790144}\Delta_{2,14,1}(2z) + \frac{5153}{930048}\Delta_{2,14,1}(6z) - \frac{1}{380160}\Delta_{2,14,2}(2z) - \frac{373}{42240}\Delta_{2,14,2}(6z) + \frac{1}{1487160}\Delta_{3,14,1}(2z) - \frac{28}{61965}\Delta_{3,14,1}(4z) + \frac{1}{1446884208}(-29t - 246)\Delta_{3,14,2}(2z) + \frac{1}{60286842}(-25t + 27786)\Delta_{3,14,2}(4z) + \frac{1}{1446884208}(29t + 1320)\Delta_{3,14,3}(2z) + \frac{1}{60286842}(25t + 29136)\Delta_{3,14,3}(4z) + \frac{1}{635256}\Delta_{6,14}(2z),$$

$$f_{20} = -\frac{23}{15694560}\Delta_{2,14,1}(2z) + \frac{55547}{5231520}\Delta_{2,14,1}(6z) + \frac{1}{297432}\Delta_{3,14,1}(2z) - \frac{16}{37179}\Delta_{3,14,1}(4z) + \frac{1}{28937684160}(-389t - 87294)\Delta_{3,14,2}(2z) + \frac{1}{452151315}(778t + 174588)\Delta_{3,14,2}(4z) + \frac{1}{28937684160}(389t - 66288)\Delta_{3,14,3}(2z) + \frac{1}{452151315}(-778t + 132576)\Delta_{3,14,3}(4z) + \frac{13}{3811536}\Delta_{6,14}(2z),$$

$$f_{21} = \frac{241}{290640}\Delta_{2,14,1}(2z) - \frac{328779}{96880}\Delta_{2,14,1}(6z) - \frac{23}{23760}\Delta_{2,14,2}(2z) - \frac{3483}{880}\Delta_{2,14,2}(6z) + \frac{1}{6120}\Delta_{3,14,1}(2z) + \frac{256}{765}\Delta_{3,14,1}(4z) + \frac{1}{44656920}(389t - 25476)\Delta_{3,14,2}(2z) + \frac{1}{5582115}(6656t + 454656)\Delta_{3,14,2}(4z) + \frac{1}{44656920}(157t + 50958)\Delta_{3,14,3}(2z) + \frac{1}{5582115}(-6656t + 95232)\Delta_{3,14,3}(4z) + \frac{14}{8823}\Delta_{6,14}(2z),$$

$$f_{22} = \frac{233}{1020395520}\Delta_{2,14,1}(z) - \frac{233}{15943680}\Delta_{2,14,1}(2z) - \frac{136449}{113377280}\Delta_{2,14,1}(3z) + \frac{136449}{1771520}\Delta_{2,14,1}(6z) - \frac{179}{583925760}\Delta_{2,14,2}(z) - \frac{179}{9123840}\Delta_{2,14,2}(2z) - \frac{7773}{7208960}\Delta_{2,14,2}(3z) - \frac{7773}{112640}\Delta_{2,14,2}(6z) - \frac{61}{119439360}\Delta_{3,14,1}(z) - \frac{61}{9953280}\Delta_{3,14,1}(2z) - \frac{61}{14580}\Delta_{3,14,1}(4z) + \frac{1}{705528299520}(49t + 362724)\Delta_{3,14,2}(z) + \frac{1}{117588049920}(-60013t - 138768)\Delta_{3,14,2}(2z)$$

(Table 2). Continued.

$$+\frac{1}{86124060}(49t+362724)\Delta_{3,14,2}(4z)+\frac{1}{705528299520}(-49t+360078)\Delta_{3,14,3}(z)$$
$$+\frac{1}{117588049920}(60013t+3101934)\Delta_{3,14,3}(2z)+\frac{1}{86124060}(-49t+360078)\Delta_{3,14,3}(4z)$$
$$-\frac{29}{235634688}\Delta_{4,14}(z)+\frac{44277}{26181632}\Delta_{4,14}(3z)-\frac{25}{57397248}\Delta_{6,14}(z)+\frac{25}{896832}\Delta_{6,14}(2z)$$
$$-\frac{215}{203046912}\Delta_{12,14,1}(z)+\frac{1327}{1122729984}\Delta_{12,14,2}(z),$$

$$f_{23}=\frac{233}{1020395520}\Delta_{2,14,1}(z)-\frac{233}{15943680}\Delta_{2,14,1}(2z)-\frac{136449}{113377280}\Delta_{2,14,1}(3z)+\frac{136449}{1771520}\Delta_{2,14,1}(6z)$$
$$-\frac{179}{583925760}\Delta_{2,14,2}(z)-\frac{179}{9123840}\Delta_{2,14,2}(2z)-\frac{7773}{7208960}\Delta_{2,14,2}(3z)-\frac{7773}{112640}\Delta_{2,14,2}(6z)$$
$$-\frac{61}{119439360}\Delta_{3,14,1}(z)-\frac{61}{9953280}\Delta_{3,14,1}(2z)-\frac{61}{14580}\Delta_{3,14,1}(4z)$$
$$+\frac{1}{705528299520}(49t+362724)\Delta_{3,14,2}(z)+\frac{1}{117588049920}(-60013t-138768)\Delta_{3,14,2}(2z)$$
$$+\frac{1}{86124060}(49t+362724)\Delta_{3,14,2}(4z)+\frac{1}{705528299520}(-49t+360078)\Delta_{3,14,3}(z)$$
$$+\frac{1}{117588049920}(60013t+3101934)\Delta_{3,14,3}(2z)+\frac{1}{86124060}(-49t+360078)\Delta_{3,14,3}(4z)$$
$$-\frac{29}{235634688}\Delta_{4,14}(z)+\frac{44277}{26181632}\Delta_{4,14}(3z)-\frac{25}{57397248}\Delta_{6,14}(z)+\frac{25}{896832}\Delta_{6,14}(2z)$$
$$-\frac{215}{203046912}\Delta_{12,14,1}(z)+\frac{1327}{1122729984}\Delta_{12,14,2}(z).$$

$$(1+2p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}}(2+p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{2}+\frac{a_6}{12}+\frac{a_{12}}{6}}$$
$$k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}}=\frac{k^{14}}{2^{b_1+b_5}}p^{b_1}(1-p)^{b_2}(1+p)^{b_3}$$
$$(1+2p)^{b_4}(2+p)^{b_5}$$
$$=k^{14}(k_0+k_1p+k_2p^2+k_3p^3+k_4p^4+k_5p^5+k_6p^6$$
$$+k_7p^7+k_8p^8+k_9p^9+k_{10}p^{10}+k_{11}p^{11}$$
$$+k_{12}p^{12}+k_{13}p^{13}+k_{14}p^{14}+k_{15}p^{15}+k_{16}p^{16}$$
$$k_{17}p^{17}+k_{18}p^{18}+k_{19}p^{19}+k_{20}p^{20}+k_{21}p^{21}+k_{22}p^{22}$$
$$+k_{23}p^{23}+k_{24}p^{24}+k_{25}p^{25}+k_{26}p^{26}+k_{27}p^{27}+k_{28}p^{28})$$

$$=\frac{c_1}{24}\left(1-24\sum_{n=1}^{\infty}\sigma_{13}(n)q^n\right)+\frac{c_2}{24}\left(1-24\sum_{n=1}^{\infty}\sigma_{13}(n)q^{2n}\right)$$
$$+\frac{c_3}{24}\left(1-24\sum_{n=1}^{\infty}\sigma_{13}(n)q^{3n}\right)+\frac{c_4}{24}\left(1-24\sum_{n=1}^{\infty}\sigma_{13}(n)q^{4n}\right)$$
$$+\frac{c_5}{24}\left(1-24\sum_{n=1}^{\infty}\sigma_{13}(n)q^{6n}\right)+\frac{c_6}{24}\left(1-24\sum_{n=1}^{\infty}\sigma_{13}(n)q^{12n}\right)$$
$$+r_1q^9\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{19}(1-q^{6n})^{13}(1-q^{12n})^7}{(1-q^{2n})^{11}}$$
$$+r_2q^9\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{14}(1-q^{6n})^{18}(1-q^{12n})^6}{(1-q^{2n})^{10}}$$
$$+r_3q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{16}(1-q^{6n})^4(1-q^{12n})^{16}}{(1-q^{2n})^8}$$
$$+r_4q^3\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{12}(1-q^{4n})^{12}(1-q^{6n})^8}{(1-q^{12n})^4}$$
$$+r_5.q^{11}\prod_{n=1}^{\infty}\frac{((1-q^{4n})^6 1-q^{6n})^{14}(1-q^{12n})^{14}}{(1-q^{2n})^6}$$

Table 3:

No	b_1	b_2	b_3	b_4	b_5	a_2	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	c_6	c_{12}
1	1	0	0	0	27	−40	80	12	−24	1	−8193	0	8192	0	0
2	1	0	1	0	25	−39	75	17	−25	”	”	”	”	”	”
3	1	0	2	0	23	−38	70	22	−26	”	”	”	”	”	”
4	1	0	3	0	21	−37	65	27	−27	”	”	”	”	”	”
5	1	0	4	0	19	−36	60	32	−28	”	”	”	”	”	”
6	1	0	5	0	17	−35	55	37	−29	”	”	”	”	”	”
7	1	0	6	0	15	−34	50	42	−30	”	”	”	”	”	”
8	1	0	7	0	13	−33	45	47	−31	”	”	”	”	”	”
9	1	0	8	0	11	−32	40	52	−32	”	”	”	”	”	”
10	1	0	9	0	9	−31	35	57	−33	”	”	”	”	”	”
11	1	0	10	0	7	−30	30	62	−34	”	”	”	”	”	”
12	1	0	11	0	5	−29	25	67	−35	”	”	”	”	”	”
13	1	0	12	0	3	−28	20	72	−36	”	”	”	”	”	”
14	1	0	13	0	1	−27	15	77	−37	”	”	”	”	”	”
15	1	1	0	1	25	−31	71	9	−21	”	”	”	”	”	”
16	1	1	1	1	23	−30	66	14	−22	”	”	”	”	”	”
17	1	1	2	1	21	−29	61	19	−23	”	”	”	”	”	”
18	1	1	3	1	19	−28	56	24	−24	”	”	”	”	”	”
19	1	1	4	1	17	−27	51	29	−25	”	”	”	”	”	”
20	1	1	5	1	15	−26	46	34	−26	”	”	”	”	”	”
21	1	1	6	1	13	−25	41	39	−27	”	”	”	”	”	”
22	1	1	7	1	11	−24	36	44	−28	”	”	”	”	”	”
23	1	1	8	1	9	−23	31	49	−29	”	”	”	”	”	”
24	1	1	9	1	7	−22	26	54	−30	”	”	”	”	”	”
25	1	1	10	1	5	−21	21	59	−31	”	”	”	”	”	”
26	1	1	11	1	3	−20	16	64	−32	”	”	”	”	”	”
27	1	1	12	1	1	−19	11	69	−33	1	−8193	$-\frac{1}{81}$	8192	$\frac{2731}{27}$	$-\frac{8192}{81}$
28	1	2	0	2	23	−22	62	6	−18	1	−8193	0	8192	0	0
29	1	2	1	2	21	−21	57	11	−19	”	”	”	”	”	”
30	1	2	2	2	19	−20	52	16	−20	”	”	”	”	”	”
31	1	2	3	2	17	−19	47	21	−21	”	”	”	”	”	”
32	1	2	4	2	15	−18	42	26	−22	”	”	”	”	”	”
33	1	2	5	2	13	−17	37	31	−23	”	”	”	”	”	”
34	1	2	6	2	11	−16	32	36	−24	”	”	”	”	”	”
35	1	2	7	2	9	−15	27	41	−25	”	”	”	”	”	”
36	1	2	8	2	7	−14	22	46	−26	”	”	”	”	”	”
37	1	2	9	2	5	−13	17	51	−27	”	”	”	”	”	”
38	1	2	10	2	3	−12	12	56	−28	”	”	”	”	”	”
39	1	2	11	2	1	−11	7	61	−29	1	−8193	$-\frac{1}{9}$	8192	$\frac{2731}{3}$	$-\frac{8192}{9}$
40	1	3	0	3	21	−13	53	3	−15	1	−8193	0	8192	0	0
41	1	3	1	3	19	−12	48	8	−16	”	”	”	”	”	”
42	1	3	2	3	17	−11	43	13	−17	”	”	”	”	”	”
43	1	3	3	3	15	−10	38	18	−18	”	”	”	”	”	”
44	1	3	4	3	13	−9	33	23	−19	”	”	”	”	”	”
45	1	3	5	3	11	−8	28	28	−20	”	”	”	”	”	”
46	1	3	6	3	9	−7	23	33	−21	”	”	”	”	”	”
47	1	3	7	3	7	−6	18	38	−22	”	”	”	”	”	”
48	1	3	8	3	5	−5	13	43	−23	”	”	”	”	”	”
49	1	3	9	3	3	−4	8	48	−24	”	”	”	”	”	”
50	1	3	10	3	1	−3	3	53	−25	1	−8193	−1	8192	8193	−8192
51	1	4	0	4	19	−4	44	0	−12	1	−8193	0	8192	0	0
52	1	4	1	4	17	−3	39	5	−13	”	”	”	”	”	”
53	1	4	2	4	15	−2	34	10	−14	”	”	”	”	”	”
54	1	4	3	4	13	−1	29	15	−15	”	”	”	”	”	”
55	1	4	4	4	11	0	24	20	−16	”	”	”	”	”	”
56	1	4	5	4	9	1	19	25	−17	”	”	”	”	”	”
57	1	4	6	4	7	2	14	30	−18	”	”	”	”	”	”
58	1	4	7	4	5	3	9	35	−9	”	”	”	”	”	”
59	1	4	8	4	3	4	4	40	−20	”	”	”	”	”	”

(Table 3). Continued.

60	1	4	9	4	1	5	−1	45	−21	1	−8193	−9	8192	73737	−73728
61	1	5	0	5	17	5	35	−3	−9	1	−8193	0	8192	0	0
62	1	5	1	5	15	6	30	2	−10	"	"	"	"	"	"
63	1	5	2	5	13	7	25	7	−11	"	"	"	"	"	"
64	1	5	3	5	11	8	20	12	−12	"	"	"	"	"	"
65	1	5	4	5	9	9	15	17	−13	"	"	"	"	"	"
66	1	5	5	5	7	10	10	22	−14	"	"	"	"	"	"
67	1	5	6	5	5	11	5	27	−15	"	"	"	"	"	"
68	1	5	7	5	3	12	0	32	−16	"	"	"	"	"	"
69	1	5	8	5	1	13	−5	37	−17	1	−8193	−81	8192	663633	−663552
70	1	6	0	6	15	14	26	−6	−6	1	−8193	0	8192	0	0
71	1	6	1	6	13	15	21	−1	−7	"	"	"	"	"	"
72	1	6	2	6	11	16	16	4	−8	"	"	"	"	"	"
73	1	6	3	6	9	17	11	9	−9	"	"	"	"	"	"
74	1	6	4	6	7	18	6	14	−10	"	"	"	"	"	"
75	1	6	5	6	5	19	1	19	−11	"	"	"	"	"	"
76	1	6	6	6	3	20	−4	24	−12	"	"	"	"	"	"
77	1	6	7	6	1	21	−9	29	−13	1	−8193	−729	8192	5972697	−5971968
78	1	7	0	7	13	23	17	−9	−3	1	−8193	0	8192	0	0
79	1	7	1	7	11	24	12	−4	−4	"	"	"	"	"	"
80	1	7	2	7	9	25	7	1	−5	"	"	"	"	"	"
81	1	7	3	7	7	26	2	6	−6	"	"	"	"	"	"
82	1	7	4	7	5	27	−3	11	−7	"	"	"	"	"	"
83	1	7	5	7	3	28	−8	16	−8	"	"	"	"	"	"
84	1	7	6	7	1	29	−13	21	−9	1	−8193	−6561	8192	53754273	−53747712
85	1	8	0	8	11	32	8	−12	0	1	−8193	0	8192	0	0
86	1	8	1	8	9	33	3	−7	−1	"	"	"	"	"	"
87	1	8	2	8	7	34	−2	−2	−2	"	"	"	"	"	"
88	1	8	3	8	5	35	−7	3	−3	"	"	"	"	"	"
89	1	8	4	8	3	36	−12	8	−4	"	"	"	"	"	"
90	1	8	5	8	1	37	−17	13	−5	1	−8193	−59049	8192	483788457	−483729408
91	1	9	0	9	9	41	−1	−15	3	1	−8193	0	8192	0	0
92	1	9	1	9	7	42	−6	−10	2	"	"	"	"	"	"
93	1	9	2	9	5	43	−11	−5	1	"	"	"	"	"	"
94	1	9	3	9	3	44	−16	0	0	"	"	"	"	"	"
95	1	9	4	9	1	45	−21	5	−1	1	−8193	−531441	8192	4354096113	−4353564672
96	1	10	0	10	7	50	−10	−18	6	1	−8193	0	8192	0	0
97	1	10	1	10	5	51	−15	−13	5	"	"	"	"	"	"
98	1	10	2	10	3	52	−20	−8	4	"	"	"	"	"	"
99	1	10	3	10	1	53	−25	−3	3	1	−8193	−4782969	8192	39186865017	−39182082048
100	1	11	0	11	5	59	−19	−21	9	1	−8193	0	8192	0	0
101	1	11	1	11	3	60	−24	−16	8	"	"	"	"	"	"
102	1	11	2	11	1	61	−29	−11	7	1	−8193	−43046721	8192	352681785153	−352638738432
103	1	12	0	12	3	68	−28	−24	12	1	−8193	0	8192	0	0
104	1	12	1	12	1	69	−33	−19	11	1	−8193	−387420489	8192	3174136066377	−3173748645888
105	1	13	0	13	1	77	−37	−27	15	1	−8193	−3486784401	8192	28567224597393	−28563737812992
106	3	0	12	0	1	−24	12	68	−28	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$
107	3	1	11	1	1	−16	8	60	−24	0	0	$\frac{1}{81}$	0	$-\frac{2731}{27}$	$\frac{8192}{81}$
108	3	2	10	2	1	−8	4	52	−20	0	0	$\frac{1}{9}$	0	$-\frac{2731}{3}$	$\frac{8192}{9}$
109	3	3	9	3	1	0	0	44	−16	0	0	1	0	−8193	8192
110	3	4	8	4	1	8	−4	36	−12	0	0	9	0	−73737	73728
111	3	5	7	5	1	16	−8	28	−8	0	0	81	0	−663633	663552
112	3	6	6	6	1	24	−12	20	−4	0	0	729	0	−5972697	5971968
113	3	7	5	7	1	32	−16	12	0	0	0	6561	0	−53754273	53747712
114	3	8	4	8	1	40	−20	4	4	0	0	59049	0	−483788457	483729408
115	3	9	3	9	1	48	−24	−4	8	0	0	531441	0	−4354096113	43535646672
116	3	10	2	10	1	56	−28	−12	12	0	0	4782969	0	−39186865017	39182082048
117	3	11	1	11	1	64	−32	−20	16	0	0	43046721	0	−352681785153	352638738432
118	3	12	0	12	1	72	−36	−28	20	0	0	387420489	0	−3174136066377	3173748645888
119	5	0	11	0	1	−21	9	59	−19	0	0	$-\frac{1}{729}$	0	$\frac{2731}{243}$	$-\frac{8192}{729}$
120	5	1	10	1	1	−13	5	51	−15	0	0	$-\frac{1}{81}$	0	$\frac{2731}{27}$	$-\frac{8192}{81}$
121	5	2	9	2	1	−5	1	43	−11	0	0	$-\frac{1}{9}$	0	$\frac{2731}{3}$	$-\frac{8192}{9}$
122	5	3	8	3	1	3	−3	35	−7	0	0	−1	0	8193	−8192
123	5	4	7	4	1	11	−7	27	−3	0	0	−9	0	73737	−73728

(Table 3). Continued.

124	5	5	6	5	1	19	−11	19	1	0	0	−81	0	663633	−663552
125	5	6	5	6	1	27	−15	11	5	0	0	−729	0	5972697	−5971968
126	5	7	4	7	1	35	−19	3	9	0	0	−6561	0	53754273	−53747712
127	5	8	3	8	1	43	−23	−5	13	0	0	−59049	0	483788457	−483729408
128	5	9	2	9	1	51	−27	−13	17	0	0	−531441	0	4354096113	−4353564672
129	5	10	1	10	1	59	−31	−21	21	0	0	−4782969	0	39186865017	−39182082048
130	5	11	0	11	1	67	−35	−29	25	0	0	−43046721	0	352681785153	−352638738432
131	7	0	10	0	1	−18	6	50	−10	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$
132	7	1	9	1	1	−10	2	42	−6	0	0	$\frac{1}{81}$	0	$-\frac{2731}{27}$	$\frac{8192}{81}$
133	7	2	8	2	1	−2	−2	34	−2	0	0	$\frac{1}{9}$	0	$-\frac{2731}{3}$	$\frac{8192}{9}$
134	7	3	7	3	1	6	−6	26	2	0	0	1	0	−8193	8192
135	7	4	6	4	1	14	−10	18	6	0	0	9	0	−73737	73728
136	7	5	5	5	1	22	−14	10	10	0	0	81	0	−663633	663552
137	7	6	4	6	1	30	−18	2	14	0	0	729	0	−5972697	5971968
138	7	7	3	7	1	38	−22	−6	18	0	0	6561	0	−53754273	53747712
139	7	8	2	8	1	46	−26	−14	22	0	0	59049	0	−483788457	483729408
140	7	9	1	9	1	54	−30	−22	26	0	0	531441	0	−4354096113	4353564672
141	7	10	0	10	1	62	−34	−30	30	0	0	4782969	0	−39186865017	39182082048
142	9	0	9	0	1	−15	3	41	−1	0	0	$-\frac{1}{729}$	0	$\frac{2731}{243}$	$-\frac{8192}{729}$
143	9	1	8	1	1	−7	−1	33	3	0	0	$-\frac{1}{81}$	0	$\frac{2731}{27}$	$-\frac{8192}{81}$
144	9	2	7	2	1	1	−5	25	7	0	0	$-\frac{1}{9}$	0	$\frac{2731}{3}$	$-\frac{8192}{9}$
145	9	3	6	3	1	9	−9	17	11	0	0	−1	0	8193	−8192
146	9	4	5	4	1	17	−13	9	15	0	0	−9	0	73737	−73728
147	9	5	4	5	1	25	−17	1	19	0	0	−81	0	663633	−663552
148	9	6	3	6	1	33	−21	−7	23	0	0	−729	0	5972697	−5971968
149	9	7	2	7	1	41	−25	−15	27	0	0	−6561	0	53754273	−53747712
150	9	8	1	8	1	49	−29	−23	31	0	0	−59049	0	483788457	−483729408
151	9	9	0	9	1	57	−33	−31	35	0	0	−531441	0	4354096113	−4353564672
152	11	0	8	0	1	−12	0	32	8	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$
153	11	1	7	1	1	−4	−4	24	12	0	0	$\frac{1}{81}$	0	$-\frac{2731}{27}$	$\frac{8192}{81}$
154	11	2	6	2	1	4	−8	16	16	0	0	$\frac{1}{9}$	0	$-\frac{2731}{3}$	$\frac{8192}{9}$
155	11	3	5	3	1	12	−12	8	20	0	0	1	0	−8193	8192
156	11	4	4	4	1	20	−16	0	24	0	0	9	0	−73737	73728
157	11	5	3	5	1	28	−20	−8	28	0	0	81	0	−663633	663552
158	11	6	2	6	1	36	−24	−16	32	0	0	729	0	−5972697	5971968
159	11	7	1	7	1	44	−28	−24	36	0	0	6561	0	−53754273	53747712
160	11	8	0	8	1	52	−32	−32	40	0	0	59049	0	−483788457	483729408
161	13	0	7	0	1	−9	−3	23	17	0	0	$-\frac{1}{729}$	0	$\frac{2731}{243}$	$-\frac{8192}{729}$
162	13	1	6	1	1	−1	−7	15	21	0	0	$-\frac{1}{81}$	0	$\frac{2731}{27}$	$-\frac{8192}{81}$
163	13	2	5	2	1	7	−11	7	25	0	0	$-\frac{1}{9}$	0	$\frac{2731}{3}$	$-\frac{8192}{9}$
164	13	3	4	3	1	15	−15	−1	29	0	0	−1	0	8193	−8192
165	13	4	3	4	1	23	−19	−9	33	0	0	−9	0	73737	−73728
166	13	5	2	5	1	31	−23	−17	37	0	0	−81	0	663633	−663552
167	13	6	1	6	1	39	−27	−25	41	0	0	−729	0	5972697	−5971968
168	13	7	0	7	1	47	−31	−33	45	0	0	−6561	0	53754273	−53747712
169	15	0	6	0	1	−6	−6	14	26	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$
170	15	1	5	1	1	2	−10	6	30	0	0	$\frac{1}{81}$	0	$-\frac{2731}{27}$	$\frac{8192}{81}$
171	15	2	4	2	1	10	−14	−2	34	0	0	$\frac{1}{9}$	0	$-\frac{2731}{3}$	$\frac{8192}{9}$
172	15	3	3	3	1	18	−18	−10	38	0	0	1	0	−8193	8192
173	15	4	2	4	1	26	−22	−18	42	0	0	9	0	−73737	73728
174	15	5	1	5	1	34	−26	−26	46	0	0	81	0	−663633	663552
175	15	6	0	6	1	42	−30	−34	50	0	0	729	0	−5972697	5971968
176	17	0	5	0	1	−3	−9	5	35	0	0	$-\frac{1}{729}$	0	$\frac{2731}{243}$	$-\frac{8192}{729}$
177	17	1	4	1	1	5	−13	−3	39	0	0	$-\frac{1}{81}$	0	$\frac{2731}{27}$	$-\frac{8192}{81}$
178	17	2	3	2	1	13	−17	−11	43	0	0	$-\frac{1}{9}$	0	$\frac{2731}{3}$	$-\frac{8192}{9}$
179	17	3	2	3	1	21	−21	−19	47	0	0	−1	0	8193	−8192
180	17	4	1	4	1	29	−25	−27	51	0	0	−9	0	73737	−73728
181	17	5	0	5	1	37	−29	−35	55	0	0	−81	0	663633	−663552
182	19	0	4	0	1	0	−12	−4	44	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$
183	19	1	3	1	1	8	−16	−12	48	0	0	$\frac{1}{81}$	0	$-\frac{2731}{27}$	$\frac{8192}{81}$
184	19	2	2	2	1	16	−20	−20	52	0	0	$\frac{1}{9}$	0	$-\frac{2731}{3}$	$\frac{8192}{9}$
185	19	3	1	3	1	24	−24	−28	56	0	0	1	0	−8193	8192
186	19	4	0	4	1	32	−28	−36	60	0	0	9	0	−73737	73728
187	21	0	3	0	1	3	−15	−13	53	0	0	$-\frac{1}{729}$	0	$\frac{2731}{243}$	$-\frac{8192}{729}$
188	21	1	2	1	1	11	−19	−21	57	0	0	$-\frac{1}{81}$	0	$\frac{2731}{27}$	$-\frac{8192}{81}$

(Table 3). Continued.

189	21	2	1	2	1	19	−23	−29	61	0	0	$-\frac{1}{9}$	0	$\frac{2731}{3}$	$-\frac{8192}{9}$
190	21	3	0	3	1	27	−27	−37	65	0	0	−1	0	8193	−8192
191	23	0	2	0	1	6	−18	−22	62	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$
192	23	1	1	1	1	14	−22	−30	66	0	0	$\frac{1}{81}$	0	$-\frac{2731}{27}$	$\frac{8192}{81}$
193	23	2	0	2	1	22	−26	−38	70	0	0	$\frac{1}{9}$	0	$-\frac{2731}{3}$	$\frac{8192}{9}$
194	25	0	1	0	1	9	−21	−31	71	0	0	$-\frac{1}{729}$	0	$\frac{2731}{243}$	$-\frac{8192}{729}$
195	25	1	0	1	1	17	−25	−39	75	0	0	$-\frac{1}{81}$	0	$\frac{2731}{27}$	$-\frac{8192}{81}$
196	27	0	0	0	1	12	−24	−40	80	0	0	$\frac{1}{729}$	0	$-\frac{2731}{243}$	$\frac{8192}{729}$

$$+r_6q^7\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{18}(1-q^{6n})^{14}(1-q^{12n})^2}{(1-q^{2n})^6}$$
$$+r_7q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{4n})(1-q^{6n})^{19}(1-q^{12n})^{13}}{(1-q^{2n})^5}$$
$$+r_8q^3\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{13}(1-q^{4n})^7(1-q^{6n})^{13}}{(1-q^{12n})^5}$$
$$+r_9q^9\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{20}(1-q^{12n})^{12}}{(1-q^{2n})^4}$$
$$+r_{10}q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{20}(1-q^{6n})^{12}(1-q^{12n})^{12}}{(1-q^{2n})^{16}}$$
$$+r_{11}q^7\prod_{n=1}^{\infty}\frac{(1-q^{2n})^8(1-q^{4n})^{20}(1-q^{12n})^{12}}{(1-q^{6n})^{12}}$$
$$+r_{12}q^9\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{12}(1-q^{4n})^{12}(1-q^{12n})^{20}}{(1-q^{6n})^{16}}$$
$$+r_{13}q^8\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{18}(1-q^{6n})^{20}(1-q^{12n})^2}{(1-q^{2n})^{12}}$$
$$+r_{14}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{20}(1-q^{6n})^6(1-q^{12n})^{12}}{(1-q^{2n})^{10}}$$
$$+r_{15}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^5(1-q^{6n})^{11}(1-q^{12n})^{11}}{(1-q^{2n})^9}$$
$$+r_{16}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{10}(1-q^{6n})^{16}(1-q^{12n})^{10}}{(1-q^{2n})^8}$$
$$+r_{17}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{11}(1-q^{6n})^{15}(1-q^{12n})^{15}}{(1-q^{4n})^{13}}$$
$$+r_{18}q^8\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{10}(1-q^{4n})^{16}(1-q^{12n})^{16}}{(1-q^{6n})^{14}}$$
$$+r_{19}q^{12}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^2(1-q^{6n})^{12}(1-q^{12n})^{18}}{(1-q^{2n})^4}$$
$$+r_{20}q^{12}\prod_{n=1}^{\infty}\frac{(1-q^{6n})^{17}(1-q^{12n})^{17}}{(1-q^{2n})^3(1-q^{4n})^3}$$
$$+r_{21}q^4\prod_{n=1}^{\infty}\frac{(1-q^{2n})^5(1-q^{4n})^7(1-q^{6n})^{19}}{(1-q^{12n})^5}$$
$$+r_{22}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{16}(1-q^{12n})^{16}}{(1-q^{2n})^2(1-q^{6n})^2}$$
$$+r_{23}q^6\prod_{n=1}^{\infty}(1-q^{4n})^{18}(1-q^{6n})^8(1-q^{12n})^2$$

$$=\delta(b_1)-\sum_{n=1}^{\infty}(c_1\sigma_{13}(n)+c_2\sigma_{13}\left(\frac{n}{2}\right)+c_3\sigma_{13}\left(\frac{n}{3}\right)+$$
$$c_4\sigma_{13}\left(\frac{n}{4}\right)+c_6\sigma_{13}\left(\frac{n}{6}\right)+c_{12}\sigma_{13}\left(\frac{n}{12}\right))q^n$$
$$+r_1f_1(n)+\ldots+r_{23}f_{23}(n),$$

where

$$\delta(b_1)=\begin{cases}0 \text{ if } b_1\neq 0\\ 1 \text{ if } b_1=0\end{cases}.$$

So

$$c(2n)=-c_1\sigma_{13}(2n)-c_2\sigma_{13}(n)-c_4\sigma_{13}\left(\frac{n}{2}\right)-$$
$$(16\,385c_3+c_6)\sigma_{13}\left(\frac{n}{3}\right)$$
$$-(c_{12}-16\,384c_3)\sigma_{13}\left(\frac{n}{6}\right)+r_{13}f_{13}(2n)+\cdots+r_{23}f_{23}(2n),$$

Therefore, for n=1,2,…,

$$c(2n)=-c_1\sigma_{13}(2n)-c_2\sigma_{13}(n)-c_4\sigma_{13}\left(\frac{n}{2}\right)-$$
$$(16\,385c_3+c_6)\sigma_{13}\left(\frac{n}{3}\right)$$
$$-(c_{12}-16\,384c_3)\sigma_{13}\left(\frac{n}{6}\right)+r_{13}f_{13}(2n)+\cdots+r_{23}f_{23}(2n),$$

$$c(2n-1)=-c_1\sigma_{13}(2n-1)-c_3\sigma_{13}\left(\frac{2n-1}{3}\right)$$
$$+r_1f_1(2n-1)+\cdots+r_{12}f_{12}(2n-1),$$

since it is easy to see that

$$\sigma_k\left(\frac{2n}{3}\right)=(2^k+1)\sigma_k\left(\frac{n}{3}\right)-2^k\sigma_k\left(\frac{n}{6}\right)$$

hence,

$$\sigma_{13}\left(\frac{2n}{3}\right)=16\,385\sigma_{13}\left(\frac{n}{3}\right)-16\,384\sigma_{13}\left(\frac{n}{6}\right),$$

and, for n=1,2,…,

$$f_1(2n)=\cdots=f_{12}(2n)=0,$$

$$f_{13}(2n-1)=\cdots=f_{23}(2n-1)=0.$$

Remark 1. We have found 196 eta quotients, see Table **3**, such that, for n=1,2,…,

$$c(2n) = -c_1\sigma_{13}(2n) - c_2\sigma_{13}(n) - c_4\sigma_{13}\left(\frac{n}{2}\right) - (16385c_3 + c_6)\sigma_{13}\left(\frac{n}{3}\right) - (c_{12} - 16384c_3)\sigma_{13}\left(\frac{n}{6}\right)$$

$$c(2n-1) = -c_1\sigma_{13}(2n-1) - c_3\sigma_{13}\left(\frac{2n-1}{3}\right) + r_1f_1(2n-1) + \cdots + r_{12}f_{12}(2n-1).$$

and 459 eta quotients, such that for n=1,2,…,

$$c(2n) = -c_1\sigma_{13}(2n) - c_2\sigma_{13}(n) - c_4\sigma_{13}\left(\frac{n}{2}\right) - c_6\sigma_{13}\left(\frac{n}{3}\right) - c_{12}\sigma_{13}\left(\frac{n}{6}\right) + r_{13}f_{13}(2n) + \cdots + r_{23}f_{23}(2n),$$

$$c(2n-1) = 0.$$

Remark 2. If f is an eta quotient, then $f(-q)$ is also an eta quotient, so the coefficients of $\frac{1}{2}(f(q) + f(-q))$ are exactly the even coefficients of f. In particular, it means that we have obtained all coefficients of some sum of 196 eta quotients.

Remark 3. $S_{14}(\Gamma_0(12))$ is 23 dimensional, $M_{14}(\Gamma_0(12))$ is 29 dimensional, see [18] (Chapter 3, pg.87 and Chapter 5, pg.197), and generated by

$\Delta_{2,14,1}, \Delta_{2,14,1}(2z), \Delta_{2,14,1}(3z), \Delta_{2,14,1}(6z),$
$\Delta_{2,14,2}, \Delta_{2,14,2}(2z), \Delta_{2,14,2}(3z), \Delta_{2,14,2}(6z),$
$\Delta_{3,14,1}(z), \Delta_{3,14,1}(2z), \Delta_{3,14,1}(4z), \Delta_{3,14,2}, \Delta_{3,14,2}(2z),$
$\Delta_{3,14,2}(4z), \Delta_{3,14,3}(z), \Delta_{3,14,3}(2z), \Delta_{3,14,3}(4z)$
$\Delta_{4,14}, \Delta_{4,14}(3z), \Delta_{6,14}, \Delta_{6,14}(2z), \Delta_{12,14,1}, \Delta_{12,14,2},$

where $\Delta_{2,14,1}, \Delta_{2,14,2}$ are the unique newforms in $S_{14}(\Gamma_0(2))$, $\Delta_{3,14,1}, \Delta_{3,14,2}, \Delta_{3,14,3}$ are the newforms in $S_{14}(\Gamma_0(3))$, $\Delta_{4,14}$, is the unique newform in $S_{14}(\Gamma_0(4))$ and $\Delta_{6,14}$ is the unique newform in $S_{14}(\Gamma_0(6)), \Delta_{12,14,1}, \Delta_{12,14,2}$ are the unique newforms in $S_{14}(\Gamma_0(12))$. By taking t as a root of $x^2 + 54x - 16992$, we express $f_1, \cdots f_{23}$ in Table **2** as linear combinations of them.

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