Modelling the Stochastic Behavior of the Pakistan Stock Exchange

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Abstract: This research study examines the stochastic behaviour of stock returns in the Pakistan stock exchange (PSX) using the KSE 100 index daily return data from 2011 to 2016. Methodology is based on ARCH family of models such as ARCH, GARCH, GARCH-M EGARCH, EGARCH-M and TARCH/GJR GARCH. These specifications have been used to model the stochastic behaviour of stock return in the Pakistan stock exchange. The most appropriate model is selected on the basis of SIC and AIC selection criteria. Diagnostic checking includes ARCH-LM test and Ljung-Box Q-Statistics. This research study concludes that EGARCH (1, 1) specification explains the stochastic behaviour of the Pakistan stock exchange better than other models undertaken. The results of the study have well-grounded policy implications for market regulators, policy makers and investors.

Keywords: Stochastic Behaviour, ARCH, GARCH, GARCH-M, TARCH, EGARCH, EGARCH-M, Pakistan Stock Exchange, Volatility.

1. INTRODUCTION

Different research studies have modelled the stochastic behaviour of stock markets in the developed economies. There exist emerging capital markets in different developing countries. Such markets attract both investors and multinational enterprises for the diversification of their assets because of global integration and also increase co-movement of different capital markets [1, 2].

Financial reforms intending to be liberalised the Pakistan stock exchange are under way. In this regard, regularity authorities are taking decisions to make the PSX more efficient and accessible to both domestic and foreign investors.

The hypothesis of market efficiency reflects the accurate prediction of stock prices containing the investor's expectations about the present value (PV) of the future cash flows. In this regard, volatility tests are inevitable for providing the evidence on market rationality and modeling the stochastic behavior of stock prices simultaneously. These tests and models include ARCH, GARCH, GARCH-M, TARCH, EGARCH and EGARCH-M.

This paper tries to investigate empirically the volatile and stochastic nature of an emerging market of Pakistan. The analysis is followed out using ARCH family of models. The objective of the study is to model the volatility and stochastic behaviour of stock returns using ARCH-type models in the Pakistan stock exchange (PSX).

Modelling the volatility or stochastic behaviour of the stock market has been an area of interest by academics and also one of the most significant concepts in Financial Economics. Volatility measured through variance and standard deviation of stock returns is utilised as a proxy of risk of common stocks. The calculation of historical volatility is based on variance or standard deviation of asset returns over the historical period undertaken. Secondly, volatility clustering of stock returns series is the basic motivation for the ARCH family of models. Volatility clustering reflects the periods of large fluctuations in stock returns are followed by the periods of large fluctuations either positive or negative. Likewise, a period of small volatility is followed by a period of small period.

All the models under consideration take into account the conditional time-varying variance. The symmetric models such as ARCH and GARCH are appropriate to capture the equity effect and asymmetric models like TARCH and EGARCH are used for leverage effect. The conditional variance of an asymmetric model (TARCH) behaves differently to the negative and positive random shocks. Besides, EGARCH process captures asymmetric behaviour of stock return i.e. leverage effect which is related to good or bad news of stochastic volatility. Tsay (2005) [3] emphasizes on the use of asymmetric model like TARCH and EGARCH because both models can capture the effect of negative shocks on the volatility more than that of positive ones. Whereas GARCH-M model hypothesized that change in conditional variance had a direct effect on expected return of a portfolio. Moreover, EGARCH-M would capture the effect of shocks on the stochastic behaviour of stock returns.

The organisation of the paper is as follows: The empirical literature is reviewed section II. Section III

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introduces different volatility models to capture the stochastic behaviour. Section IV describes data and methodology. The results are discussed in section V. Section VI concludes the study.

2. LITERATURE REVIEW

Stochastic behaviour of stock prices is of great important in financial economics. The key reason is the movements of stock returns. Different researchers have examined the stochastic behaviour using different models belonging to ARCH family. These empirical studies are as follows:

Engle (1982) [4] examines the time-varying behaviour of stock returns using ARCH model. Unlike the assumption of constant variance, the study finds that conditional variance is changing over time and is a function of previous errors. Empirically the ARCH model is used with some limitations including long lag in the conditional variances and a fixed lag structure is used to overcome the problem of negative variance parameters.

Bollerslev (1986) [5] introduces GARCH model to avoid the ARCH limitations. GARCH model allow for both a long and a flexible lag structure. GARCH model includes lagged conditional variances as compared to ARCH model which allows the conditional variance is a linear function of past variance.

The ARCH-M model is proposed by Engle, Lilien and Robins (1987) [6]. This model takes the conditional variance as a part of the mean equation of asset returns. The conditional variance is a linearly related to the previous squared innovations in a simple ARCH model. However, it is hypothesized in ARCH-M model that the mean return of the portfolio is directly affected by the time-varying conditional variance (risk). Their findings indicate that risk premia are time variant systematically with respect to the investor's expectation about the inherent uncertainty.

Nelson (1991) [7] proposed EGARCH model to examine the stochastic behaviour of returns. It hypothesized that positive and negative excess returns responded differently to variances of stock returns. The finding indicated that not only hypothesis proved, but also found that variance and excess returns were negatively related to each other. On the other hand EGARCH-M captured the asymmetric effect of shocks on volatility and used for risk premium.

Glosten, Jagannathan, and Runkle (1993) [8] propose TARCH (known as a GJR GARCH model)

model as a result of modification of GARCH-M specification on the basis of GARCH model which explains a symmetric response of variance to both type of shocks i.e. positive and negative. The study finds that both the conditional mean and conditional variance of excess returns are positively and significantly related in case of the GARCH-in-Mean model. After the modification of the GARCH-M specification, the study indicates conditional variances react differently to both positive and negative unanticipated return which also behaves oppositely to future conditional variances.

Nishat (2000) [9] observes a significant increase in risk premia during the reform period in the Karachi Stock Exchange (KSE). The study highlights that institutional development and financial liberalization bring more players, more securities and high quality information in risk diversification that destabilize security prices, thereby increase in volatility. The change in returns has been observed during both non-reform and reform periods, also increases the market volatility. The volatility in returns includes both the volatility of the stock and volatility induced by the changes in government policies. The GARCH-M model has been used to capture the stochastic and volatility behavior of the Pakistan stock exchange. The GARCH-M process highlights higher compensation for risk means higher average return in KSE after liberalization.

Mahmud and Mirza (2011) [10] explore the stochastic behaviour of stock returns using ARCH-type models such as the GARCH, EGARCH and GJR-GARCH in the Karachi stock exchange. The sample size consists of 6-year from 2004 to 2009. The study concludes that the EGARCH (1, 1) model is appeared as the most efficient one among different models to capture the asymmetric effect for an emerging market of Pakistan during crisis.

Waqar (2014) [11] models the volatility using the univariate models of heteroskedasticity to capture both symmetric (equity) and asymmetric (leverage) effects in the Karachi Stock Exchange (KSE). Both effects have been analysed from 4th of January 2010 to 18th of May 2015 using daily stock return series of KSE-100 index. The findings of this research study confirm the adequacy of both EGARCH and TARCH models in the context of the Karachi Stock Exchange (KSE) to capture the leverage (asymmetric) effect.

3. MODELS FOR STOCHASTIC BEHAVIOR

These models include both symmetric and asymmetric models to capture the stochastic behavior of the Pakistan stock exchange (PSX).

3.1. Symmetric Models

The symmetric models include both ARCH and GARCH models. Engel (1982) [4] introduces ARCH model to capture the movements in financial series. The ARCH model is based on heteroskedasticity (the variance of the error term is not constant). The conditional variance u_t can be denoted by σ_t^2, which depends on the past values of the squared error,

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{1}$$

The equation (1) is known as an ARCH (1). Under ARCH, the conditional mean equation would be

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + \mu_t \quad \mu_t \sim N(0, \sigma_t^2) \tag{2}$$

The equation (1) can be generalized up to q lags of squared errors, which would be known as an ARCH (q) model:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \alpha_3 u_{t-3}^2 ..... + \alpha_q u_{t-q}^2 \tag{3}$$

The GARCH model, which was proposed by Bollerslev (1986) [5] and Taylor (1986) [12], modifies the ARCH model in equation (1) by taking the previous lags of the conditional variance as independent variables. As a result, GARCH (1, 1) model takes the following form:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{4}$$

The GARCH (1, 1) specification can be generalized up to a GARCH (p,q) formulation which takes the following form:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + ... + \alpha_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_{p1} \sigma_{t-p}^2 \tag{5}$$

$$\sigma_t^2 = \alpha_0 + \sum \alpha_i u_{t-i}^2 + \sum \beta_j \sigma_{t-j}^2 \tag{6}$$

Generally, the GARCH (1, 1) model is considered sufficient to capture the stochastic and volatility behavior of the data [13].

3.2. Asymmetric Models

One of limitations of GARCH models is that they respond to positive and negative shocks symmetrically. However, it is argued that a negative shock behaves differently to financial time series as compared to a positive shock, such asymmetries are known to be as *leverage effects*. Two well-known asymmetric models are known as: the GJR model and the exponential GARCH (EGARCH) model.

The GJR/TARCH model is proposed by Glosten, Jagannathan and Runkle (1993) [8]. TARCH is an extended version of GARCH to account the asymmetric behaviour of stock returns. The conditional variance is as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \mu_{t-1}^2 + ... + I_{t-1} \tag{7}$$

To find a leverage effect, γ would be positive ($\gamma > 0$). The EGARCH model was introduced by Nelson (1991) [7]. The specification of EGARCH model for the conditional variance equation takes the following form:

$$In(\sigma_t^2) = \omega + \beta In(\sigma_{t-1}^2) + \gamma \frac{\mu_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|\mu_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{8}$$

The exponential GARCH specification has many advantages. First, the $In(\sigma_t^2)$ indicates that σ_t^2 appears positive even in the case of negative parameters. Moreover, the exponential GARCH specification allows for asymmetries, although volatility and returns are negatively related, γ, will behave negatively. Hence, the EGARCH model does not require artificial non-negativity constraints on the parameters.

GARCH-M specification is an extended form of an ARCH-M model proposed by Engle, Lilien and Robins (1987) [6]. GARCH-M model takes the following form:

$$y_t = \mu + \delta \sigma_{t-1} + \mu_t, \quad \mu_t \sim N(0, \sigma_t^2) \tag{9}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{10}$$

The risk premium is indicated by δ which is related to the conditional variance that ultimately affects the mean of average return. EGARCH-M model proposed by Nelson (1991) [7] is also used to estimate the risk premium.

4. DATA AND METHODOLOGY

The daily return data on KSE 100 index is retrieved from www.finance.yahoo.com from 2011 to 2016. The returns (R) of KSE-100 index at time (t) are calculated in the log form of KSE-100 index (p) can be defined as: $R_t = \{ ln (P_t/P_{t-1})\}$

4.1. Unit Root Analysis

The unit root analysis contains both the Augmented Dickey Fuller test and the Phillips-Perron test to analyse the stationarity of the stock return series.
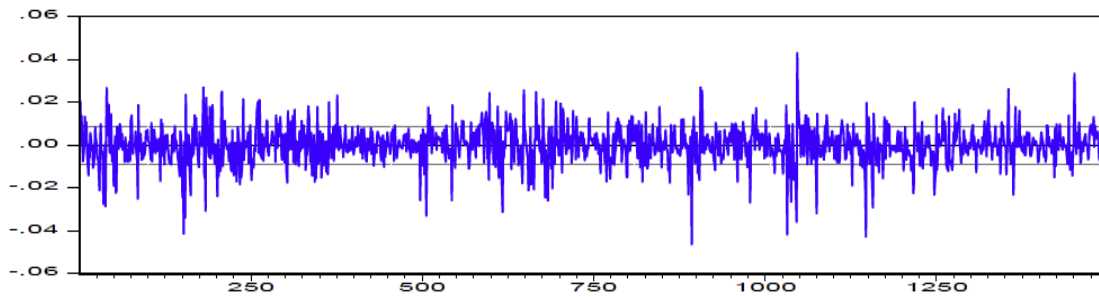
Figure 1: Clustering Volatility.

4.1.1. The Augmented Dickey Fuller (ADF) Test

ADF test is used to test the null hypothesis of non-stationarity. It takes the following form:

$$\Delta Y_t = \beta_1 + \beta_{2t} + \alpha_0 Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \varepsilon_t$$

The null hypothesis can be rejected as α0 is appeared significant statistically.

4.1.2. The Phillips-Perron Test

PP is also used to test the stationarity of return series in this study. It proceeds as:

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \beta_3(t - T/2) + \varepsilon_t$$

Where β's are parameters, T is total number of observations, t is time trend and ε_t is error term. For unit root null hypothesis is H_0: $\beta_2 = 1$.

4.2. Analysis of Heteroskedasticity

Two things are important to run the ARCH family models that include clustering volatility and ARCH-LM Test.

Graphical representation of residuals of daily stock returns series of KSE-100 indicates the fluctuation of returns around zero line in Figure **1**. Volatility remains low for a particular time periods and high for remaining periods. A strong autocorrelation in squared returns is shown by volatility clustering which can be explained as the periods of low volatility are followed by low periods for a prolonged period. Again the period of high volatility tends to be followed by high volatility. When this happened for residuals, it provided all justification to run ARCH family of models. But ARCH-LM test of heteroskedasticity would be appeared decisive, in this context, to run ARCH model. The findings of ARCH-LM test in Table **1** indicate that there exists ARCH effect in the residuals of mean equation of KSE-100 as the values of both F-statistics and observed R^2 appear statistically significant. It also paves the way to run both

symmetric and asymmetric models belong to ARCH family.

Table 1: ARCH-LM Test

F-Statistics	42.48	0.000
Obs*R2	41.36	0.000

The ARCH-type models have been employed through the ordinary least square method (OLS) for estimation of volatility and stochastic behaviour estimated for the series of KSE-100 returns under the assumption of the Gaussian standard normal distribution. To choose the best fitted model for the KSE-100 index data, selection criteria such as Akaike information criterion (AIC) and Schwarz information criterion (SIC) have been used. Residual diagnostic test statistics including both autoregressive conditional heteroskedasticity- Lagrange multiplier (ARCH-LM) test and correlogram squared residuals (Q-Statistics) have been employed for the selection of the best volatility model, which representing stochastic trend the conditional variance of stock returns. All the models are estimated through E-views software.

5. FINDINGS

The augmented Dickey- Fuller (ADF) and Phillip-Perron tests are used to examine whether a time series is stationary at levels or at difference because non-

Table 2: Unit Root Analysis of Stock Returns

Unit Root Tests	Level	Prob.	Remarks
ADF Test	-16.29(4)*	0.000	I(0)
PP Test	-33.64(5)*	0.000	I(0)

Level of significance is shown by *, **, *** at 1%, 5% and10 % respectively. Figures in parentheses () in third row and second column indicate lag length determined by Akaike Information Criterion (AIC) in ADF Test. Figures in parentheses () in fourth row and second column indicate Bandwidth based on Newey-West using Bartlett kernel in PP test. Variable is taken in logarithmic form.

stationary series of variable leads to weak findings. Hence, the Augmented Dicky-Fuller (ADF) and Phillip-Perron (PP) tests have been employed to determine the stationarity of the stock return series in this study. Table given above depicts that both the ADF and PP tests reject the null hypothesis of root unit at 1% level of significance for daily return series which is stationary at levels.

Table 3: Summery of Statistics

Statistics	Stock Prices	Stock Returns
Mean	24678	0.001
Median	25603	0.001
Maximum	47806	0.044
Minimum	10842	-0.045
Std.Deviation	9928	0.008
Skewness	0.093	-0.410
Kurtosis	1.721	6.009
Jarque-Bera	104.0	605.8
Probability	0.000	0.000
Observations	1495	1494

Table **3** presents summery of statistics of daily returns data. The data of 1494 days have been used in this analysis. The average mean return is 0.001 which lies between maximum average return of 0.044 and minimum of average return of -0.045. The volatility is shown by standard deviation i.e. 0.008. An asymmetric distribution of return series of KSE-100 index is appeared by negative skewness suggesting that values of KSE-100 index are declining more than increasing. The values of both skewness and kurtosis (i.e. >3) indicate that distribution has long left and fatter tails as compared to normal distribution. Moreover, the higher

value of kurtosis is also found by Nishat (2000) [9], Hafeez (2014a, 2014b) [14, 15], Hafeez and Nishat (2014) [16] and Mahmud and Mirza (2011) [10] suggesting that big surprises either positive or negative are more likely to be observed in the KSE. However, the high value of Jarque-Bera i.e. 605.8 with p-value =0.000 rejects the normality assumption.

The best model is selected for the KSE100 index to predict stochastic behaviour of the Pakistan stock exchange on the basis of minimum SIC and AIC criteria. The results given in Table **4** indicate six models of heteroskedasticity estimated to capture the stochastic volatility. The estimated models include ARCH (5), GARCH (1, 1), GARCH-M (1, 1), TARCH (1, 1), EGARCH (1, 1) and EGARCH-M (1, 1).

The most appropriate model is EGARCH (1, 1) because it has the minimum SIC and AIC values as compared to other estimated models. This finding is similar to that of Mahmud and Mirza (2011) [10]. Hansen and Lunde (2005) [17] also emphasize that the asymmetric models with leverage effects (e.g. EGARCH) can better predict return volatility than that of GARCH. In addition, EGARCH-M (1, 1) model is found as the second best model for measuring risk premium which is followed by TARCH (1, 1) model to capture leverage effect for the Pakistan stock exchange (PSX). The results of the study are also supported by Waqar (2014) [11].

Residuals diagnostic testing shows no serial correlation is found till 36th lags in the models undertaken. Table **4** also indicates that no ARCH effect is detected in the models through ARCH-LM test.

Finally, future research could be to investigate the volatility and stochastic behaviour of the Pakistan stock exchange under the impact of institutional development to confirm market stability and efficiency.

Table 4:   ARCH Models

Models	Selection Criteria		ARCH-LM Test				LB^2 Q-Stat (36)	
	AIC	SIC	F-Stat	Prob.	Obs*R^2	Prob.	Q-Stat	Prob.
ARCH (5)	-6.730	-6.705	0.186	0.666	0.186	0.666	26.99	0.861
GARCH (1,1)	-6.737	-6.721	0.057	0.811	0.057	0.811	26.32	0.881
GARCH-M(1,1)	-6.739	-6.721	0.042	0.837	0.0424	0.837	24.36	0.880
TARCH (1,1)	-6.774	-6.756	0.067	0.794	0.067	0.794	26.36	0.880
EGARCH-M(1,1)	-6.789	-6.767	0.927	0.336	0.928	0.335	27.36	0.849
EGARCH(1,1)	-6.790	-6.773	0.814	0.366	0.815	0.366	26.61	0.873

Level of significance is shown by *, **, *** at 1%, 5% and10 % respectively.

6. CONCLUSION

This study tries to model the stochastic behaviour of stock returns of the Pakistan stock exchange from 2011 to 2016. The analysis is based on both types of models such as symmetric and asymmetric. Symmetric models include ARCH and GARCH. On the other hand, TARCH, GARCH-M, EGARCH and EGARCH-M are known as asymmetric models. Pictorial representation of clustering volatility and significant ARCH effect provide the basis to run ARCH family of models. Among the different models, EGARCH (1, 1) model is appeared as a best fitted model to the data of stock returns based on minimum of SIC and AIC. This is the most appropriate model to capture the stochastic behaviour of the Pakistan stock exchange. The findings of this paper will benefit the potential users such as investors, market regulators and policy makers.

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