Relation between Luminosity and Surface Rotation of Spotted Stars

Ingila Rahim[1,*] and Muhammad Rashid Kamal Ansari[2]

[1]*Federal Urdu University of Arts Science and Technology, Karachi, Pakistan*

[2]*Sir Syed University of Engineering and Technology, Karachi, Pakistan*

Abstract: This communications explores the existence of a possible relationship between Luminosity and surface rotation for the study of the evolution of spotted stars using the data of Kepler's and DI spotted stars. For the determination of such a relationship between luminosity and rotation the dependency of rotational shear on effective temperature is to be reviewed first. The strong dependence of rotational shear on the effective temperature in the range of 3000K and 6000K is confirmed by a power law. This dependence in turn introduces rotation as an evolutionary parameter for the study of the evolution of spotted stars. Multivariate Linear regression, Log-Log multivariate and Nonlinear Multivariate (2, 2) Degree models are constructed to determine the Luminosity of Kepler's and Doppler imaging spotted stars with rotational shear, relative differential rotation and radius as independent variables. In this regard Log-Log model and Nonlinear Multivariate (2, 2) Degree model is best suited as compared to the linear model. In the next stage Log-Log model is applied to the main sequence Kepler's stars (excluding giants) and also to the stars in the individual spectral classes A, F, G, K, and M. The model appears best for main sequence stars and also for the stars in the individual classes F-M. Applying the model on DI spotted stars the standard errors indicate that the adequacy of the model for DI spotted stars data is weak. A description of stellar motions and description of data and model used is given in the introduction.

Keywords: Stellar spots, Stellar Evolution, Sunspots, Differential Rotation, Kepler's Stars, Doppler Imaging stars.

1. INTRODUCTION

Sunspots are the cool, dark strong magnetic areas on solar photosphere. Like sun other cool stars also have spots called Starspots [1, 2]. Starspots are created by magnetic field and high energy and temperature gradient produced by the nuclear reactions in the stars. Stars generate energy and temperature by nuclear burning reactions in the center and vary from the center to the surface. According to theory developed by Gurevich and Lebedinskii (1946) [3] the circular motion of plasma under the convection zone intensifies the weak magnetic fields of the Sun [4,5]. When magnetic flux tubes are stressed due to differential rotation of radiative core and convective envelop of the Sun and approach a certain limit they curl like a rubber band that exerts force and temperature gradient. This force increases pressure without gaining density that creates magnetic buoyancy towards the magnetic pole. Sunspots appear at the poles of the magnetic field [6]. By sunspots observation Christoph Scheiner (1630) found that the solar surface is differentially rotating with different periods. It rotates fast at the pole and slows at the equator. Solar dynamo is the base of the theory of differential rotation. The interplay of solar dynamo and solar convection process are the key of various solar activity phenomena like sunspots. Similarly many spotted stars that are cool with convective envelope or fully convective structure were analyzed by one or many techniques. Their differential rotation was also analyzed. The recent studies reveal the relation of stellar rotation with stellar interior and evolution of stars [5-7]. Skumanich 1972 [8] was the first to find the links between the stellar rotation and age. It decreases as the star age increases [9,10]. Stellar evolution refers to the changes that take place in stars as they age (life cycle of the stars). It is the process by which a star undergoes a sequence of radical changes during its life time. In the processes of stellar evolution all stars radiate light and other form of energy.

Variation of temperature and radius are the most essential factors in the evolution process and affect stellar properties like brightness, luminosity, color and mass etc. Luminosity of a star is its apparent brightness usually measured in terms of the luminosity of the Sun (considered to be 3.85 × 10^{26} watts). Radiative diffusion restricts the flow of radiation from the sun and so that all the heat cannot be consumed rapidly it determined the luminosity and rate of relief of energy by thermos nuclear fusion. Because solar luminosity is supplied by a chain of thermonuclear reactions proton-proton chain the most remarkable reactions are as follows.

$$p+p \rightarrow d+e^{+}+\nu_e$$

$$p+d \rightarrow {}^{3}He+\gamma$$

$$^{3}He+{}^{3}He \rightarrow {}^{4}He+p+p$$

*Address correspondence to this author at the Mathematical Sciences Research Center, Federal Urdu University of Arts, Sciences & Technology, Main University Road, Karachi 75300, Pakistan; E-mail: ingila.rahim@gmail.com

ISSN: 1814-8085 / E-ISSN: 1927-5129/18

One of the important problems in the study of stellar evolution is that how different parameters can be related to establish a simple model. Hertzsprung-Russell diagram (HRD or HR Diagram) is a powerful and significant tool for the understanding of stellar evolution and the theories already developed [15]. Differential rotation is a strong parameter to identify and study the spotted [16]. This means that an HR diagram of spotted or differentially rotating stars with differential rotation as evolution parameter is needed. This study attempts to do so. DR of the stars depends on the heliographic latitude φ which decreases as the latitude increases. Differential Rotation of stars at different latitude is expressed by the following equation,

$$\Omega = \Omega_{eq} - \Delta\Omega \mathrm{Sin}^2\varphi$$

where, Ω_{eq} = rate of rotation at the equator,

$\Delta\Omega = \Omega_{eq} - \Omega_{pole} =$ difference between rates of rotation of pole and the rate of rotation of the equator. This difference is also called Rotational Shear (RS) or Absolute Shear, $\alpha = \frac{\Delta\Omega}{\Omega_{eq}}$ = Relative Differential Rotation (RDR) or Relative Shear and φ = heliographic latitude measured from the equator. $\alpha > 0$ for the stars with sun like DR, $\alpha < 0$ for stars having anti-solar DR and $\alpha = 0$ for solid body rotation or rigid body. This study is concerned with the stars having sun like DR or spots on the surface of the sun ($\alpha > 0$).

Stellar dynamo is a strong mechanism of magnetic field generation. It is the main source of Starspots formation and internal change in stars [13]. Section 2 discusses the relationship of DR with evolutionary parameters and uses it in the formation of 3D diagram of spotted stars. In section 3 the statistical model of HR diagram parameters (L,R and T) and DR is presented.

2. DEPENDENCY OF ROTATION ON EFFECTIVE TEMPERATURE

The study shows that the spotted stars with DR were found in red giant and main sequence evolutionary track. The main sequence stars have sun like DR and structure (convective envelope).They belong to F, G, K and M spectral classes. The giant stars belong to G and K spectral class [17]. The luminosity of spotted stars ranges between $0.01L_{sun}$ to 100 $L_{sun.}$. Their effective temperature is about 7000 K to 3000 K. Though, the sun like differential rotation is also found in the A spectral class their radiative core is unstable or undefined to form spots. Their relative differential rotation is very low (less than 0.1) therefore spots are failed to be detected on these hotter stars. So, on the main sequence of HR diagram, spotted stars lie in F, G, K and M spectral classes. Reinhold, T, *et al*. 2013 found that α increases with the rotation period of the star and slightly decreases with effective temperature. Whereas, $\Delta\Omega$ slightly increases between Teff=3500−6000 K. Above 6000 K a large scattering is observed. It is the location of main sequence where massive red giant spotted stars (approx. 2.5 solar mass) enter the main sequence. Doppler imaging (DI) can be used to study sunspots and differential rotation of Sun at different latitudes [17]. In addition to Doppler imaging other techniques like, photometric techniques and Astro-Seismology are also used to study spotted stars [18]. Table **1** gives the measurements of observed Rotational Shear ($\Delta\Omega$) of some stars by Doppler Imaging. Their effective temperature and radius with references are also mentioned.

There also exists a strong relationship of $\Delta\Omega$ with T_{eff} by power law $\Delta\Omega \propto T_{eff}^{p}$ L. A Balona *et al*. 2011 determined p to be 6.4, 3.5 1.0 and -2 for K, G, F and A spectral classes by an interpolation formula[19]. For the values given in the Table **1** tested and the relationship appears as $\Delta\Omega \propto T_{eff}^{7.1493}$. Barnes *et al*. 2005a [12] found the approximate value of p as 8.9. But all the previous techniques and models give strong relation between rotational shear and effective temperature. The RS increases with temperature or vice versa.

The data of DI spotted stars are too small for some appropriate statistical modeling. So data of approximately 12000 active stars in the Kepler field derived from Q3 data is used. Their absolute rotational shear and relative rotational shear was calculated by Reinhold, T. *et al*. 2013. Other evolutionary parameters data is obtained from Kepler's exoplanetary data center. The stars consist of 334 "K" spectral class giant stars and 11687 main sequence stars. The main sequence stars consist of A, F, G, K and M spectral class.

3. NON-LINEAR MODELS

The basic HR diagram of stars is an expression of the graph between their luminosity and temperature. The basic formula used to calculate the luminosity of the stars is

$$L = 4\pi R^2 \sigma T_{eff}^4 \quad (1)$$

Or $L \propto R^2 T^4$

Where, R is the radius, T_{eff} the effective temperature and σ the Stephen Boltzmann constant.

Table 1. 1 - Marsden *et al.* 2004; 2- Donati *et al.* 2000; 3 – Barnes *et al.* 2000; 4—Collier Cameron & Donati 2002; 5- Baranes *et al.* (2005a); 6-Donati *et al.* (2003); 7- Baranes *et al.* (2005b); 8- Baranes *et al.* (2004); 9- Jeffers & Donati (2009); 10- Jarvinen *et al.* (2015); 11- Koveri *et al.* 2014; 12- Fares *et al.* 2012; 13- Koveri *et al.* 2011; 14- Waite *et al.* 2011; 15- Marsden *et al.* 2011b; 16- Dunstone *et al.* 2008

Star	T_{eff} K	ΔΩ rad/d^{-1}	R_{sun}	References
HD307938	5859	0.025±0.015	1.2	1
LQ Lup	5729	0.140±0.010	1.22	2
PZ Tel	5448	0.101±0.02	1.4	3
AB Dor	5386	0.091±0.007	1.1	4
HD197890	4989	0.032±0.002	1.2	5
LQ Hya	5019	0.194±0.022	1	6
LO Peg	4577	0.036±0.007	1	7
HK Aqr	3697	0.005±0.009	0.45	8
EY Dra	3489	0.000±0.003	0.6	5
HD171488	5800	0.340±0.040	1.1	9
AF Lep	6100	0.259±0.019	1.2	10
IL HYa	4500	0.035±0.003	8.1	11
HD179949	6160	0.216±0.061	1.26	12
V889 Her	5750	0.042	1.15	13
HD106506	5900	0.240±0.030	2.15	14
HD141943	5850	0.450±0.080	1.6	15
HD155555A	5400	0.143±0.008	1.45	16
HD155555B	5050	0.088±0.006	1.45	16

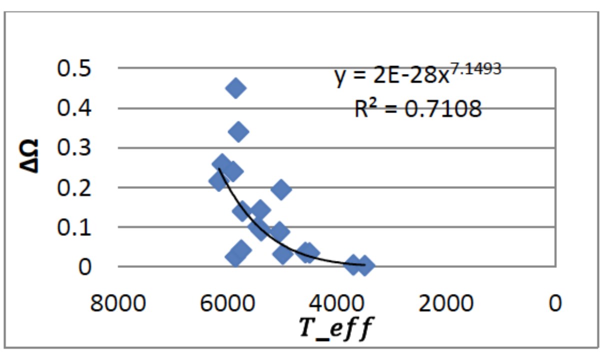

Figure 1: Show relation of T_{eff}. vs ΔΩ of Table **1** Doppler Imaging spotted stars $\Delta\Omega \propto T_{eff}^{7.1493}$.

In this study a new statistical models is form by using the parameters in luminosity formula (luminosity, radius and effective temperature). As the relation of ΔΩ with T_{eff} by power law is given as $\Delta\Omega \propto T_{eff}^{p}(\text{or } T_{eff} \propto \Delta\Omega^{q})$, (ΔΩ is used instead of Teff.

Multiple linear regression model (least square) and Log-Log multivariable regression models are applied to the data for finding the relation between evolutionary parameters, Luminosity and Rotational shear of the stars. The model equations appear as follows.

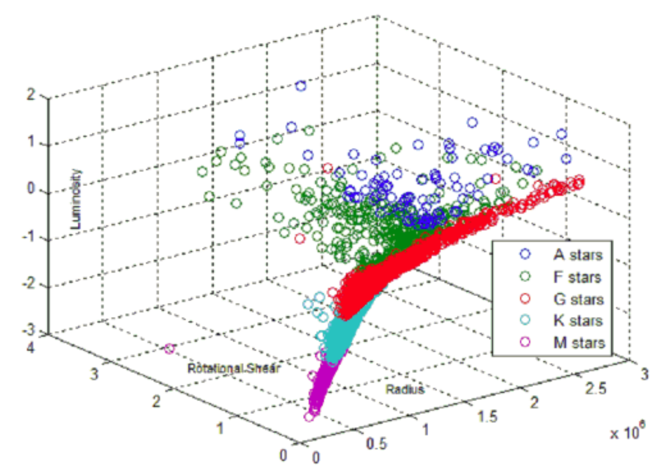

Figure 2: 3D diagram of Luminosity vs Rotational shear and Radius illustrating the relationship of parameters in different spectral classes.

$$L_{stars} = c_0 + c_1 R + c_2 \Delta\Omega \tag{2}$$

$$\log(L) = C + a \log R + b \log \Delta\Omega \tag{3}$$

Table 2: Log-Log Regression Model of Rotational Shear (Equation 3)

	C	A	b	Standard error	$<L_{star} - L_{model}>$	
					Min.	Max.
All Stars	-1.696	3.205	0.095	0.1665	-0.82	0.40
Main sequence	-3.951	3.458	0.061	0.1445	-0.92	-0.94
F Class	5.028	2.459	0.059	0.0538	-0.26	0.24
G Class	7.979	2.102	0.0195	0.0592	-0.151	0.12
K Class	-6.412	3.728	0.0259	0.0995	-0.28	0.298
M Class	2.335	2.668	-0.007	0.0321	-0.07	0.155

The models are formed using a data of approximately 11000 stars taken out of the total data of 12000 stars. The remaining data of 1000 stars is used for testing the models. The best fitted model comes out to be the multiple Log-Log regression models.

$$\log(L) = 1.696 + 3.205.\,Log\ R + 0.095.\,LOG\ \Delta\Omega \tag{4}$$

The data is found to be strongly correlated with r^2 = 0.90% and 0.1665 standard error at 95 % coefficient interval. The model is also adequate for individual main sequence, F, G K and M spectral classes of stars giving more accuracy. The results are shown in Table **2**.

Modeled stars follow the same evolutionary track in HRD as DI spotted stars. A negative correlation also appeared between relative differential rotation (α) and effective temperature. Multiple linear regression model and Log-Log multiple regression models are also used to find relations between Luminosity and Relative Differential Rotation of the stars. But the best fitted model is the multiple Log-Log regression models. The model equations appear as follows.

$$\log(L) = C + a\log R + b\log\alpha \tag{5}$$

$$\log(L) = 2.312269 + 3.254165\log R + 0.038444\log\alpha \tag{6}$$

Similarly above, the Log-Log model of relative differential rotation also give the good fit in main sequence and F, G, K and M spectral classes. Their respective accuracy and standard errors of multivariate log-log regression model between luminosity vs radius and relative differential rotation are given in Table **3**.

The both log-log models further improved by models of multivariate polynomial (2, 2) regression equation for rotational shear

$$\text{Log (L)}=C+ a\ \text{Log(R)}+b\ \text{Log}(\Delta\Omega) + c(\text{Log(R)})^2 + d\ \text{Log}(\Delta\Omega)^2+e\ \text{Log(R)}*\text{Log}(\Delta\Omega) \tag{7}$$

$$\text{Log (L)} = -191.344 +45.95\ \text{Log (R)}- 2.117\ \text{Log}(\Delta\Omega)-2.407(\text{Log(R)})^2 + 0.0274(\text{Log}(\Delta\Omega))^2 + 0.2595(\text{Log(R)}*\text{Log}(\Delta\Omega)) \tag{8}$$

and relative differential rotation.

$$\text{Log (L)} = C+ a\ \text{Log(R)} +b\ \text{Log}(\alpha) + c(\text{Log(R)})^2+d(\text{Log}(\alpha))^2+e(\text{Log}(\alpha))*\ \text{Log(R)} \tag{9}$$

$$\text{Log(L)}=-173.8+ 42.22\text{Log(R)} - 0.03\ \text{Log}(\alpha) - 2.2124(\text{Log(R)})^2-0.00309(\text{Log}(\alpha))^2-0.00122\ \text{Log}(\alpha)*\ \text{Log(R)} \tag{10}$$

The model was tested by over fitted data testing Akaike Information Criterion and Bayesian information

Table 3: Log-Log Regression Model of Relative Differential Rotation Equation (Equation 5)

	C	a	b	Standard error	$<L_{star} - L_{model}>$	
					Min.	Max.
All Stars	-2.312	3.254	-0.038	0.169	-0.82	0.42
Main sequence	-4.403	3.495	-0.034	0.146	0.384	0.387
F Class	3.790	2.590	0.002	0.0599	-0.27	0.22
G Class	7.964	2.099	-0.012	0.059	-0.519	0.12
K Class	-6.626	3.744	-0.029	0.099	-0.26	0.31
M Class	2.330	2.681	-0.003	0.032	-0.07	0.157

Table 4: Nonlinear Multivariate (2, 2) Degree Model of Rotational Shear (Equation 7)

	C	A	b	c	d	e	Standard error	$<L_{star} - L_{model}>$	
								Min.	Max.
All Stars	-191.34	45.95	-2.12	-2.41	0.03	0.26	0.13	-0.27	1.35
Main sequence	-159.68	38.76	-0.99	-1.999	0.023	0.13	0.12	-0.55	1.0266
F Class	-71.92	19.45	-1.47	-0.94	0.03	0.18	0.05	0.0095	0.524
G Class	-73.35	20.3	-0.038	-1.02	0.01	0.01	0.06	-1.01	1.547
K Class	-129.77	32.44	1.51	-1.67	-0.03	-0.18	0.099	-0.382	0.21
M Class	64.63	-12.29	-0.71	0.899	0.006	0.086	0.03	-0.068	0.174

Table 5: Nonlinear Multivariate (2, 2) Degree Model of Relative Differential Rotation (Equation 9)

	C	A	b	c	d	e	Standard error	$<L_{star} - L_{model}>$	
								Min.	Max.
Full Stars	-173.80	42.22	-0.03	-2.21	-0.003	-0.001	0.1366	-0.48	1.101
Main sequence	-145.1	35.49	-0.21	-1.82	-0.003	0.019	0.1225	-0.44	1.0094
F Class	-59.78	16.66	-0.86	-0.78	0.04	0.12	0.06	-0.281	0.181
G Class	-73.25	20.25	-0.25	-1.01	-0.0003	0.03	0.056	-0.178	0.152
K Class	-151.88	36.8	-3.57	-1.88	-0.01	0.41	0.098	-0.268	0.325
M Class	64.12	-12.06	-0.21	0.88	-0.02	0.02	0.03	-0.0913	0.145

criterion or Schwarz Criterion [13, 14]. The results are shown in Tables **4** and **5**. All the models were tested on remaining data of 1000 stars is used for testing the models. The maximum and minimum difference between the luminosity calculated by formula 1 and modeled equations also mention below tables.

4. MODEL ADEQUACY FOR DI SPOTTED STARS

In this section adequacy of the multivariate Log-Log model and Nonlinear Multivariate (2, 2) Degree model for RS of stars developed in section 3 will be tested for DI spotted stars as mentioned in Table **1**. The equations satisfies the given stars data with the standard error of 0.52972 and 0.2612 respectively. Thus the accuracy is much lower than the accuracy for Kepler's stars and the adequacy of the model for DI stars is lower as compared to the adequacy of model for Kepler's stars.

It is observed that the star IL HYa is behaving abnormally adding a large error in the model. The model error was 0.28 when IL HYa star was included in the model.

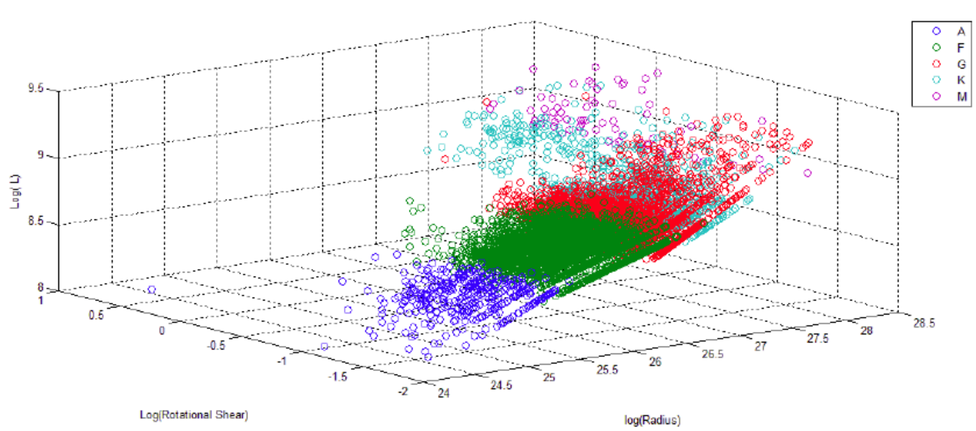

Figure 3: 3D of main sequence (A, F, G, K and M) Kepler's stars (Log RS vs Log L).

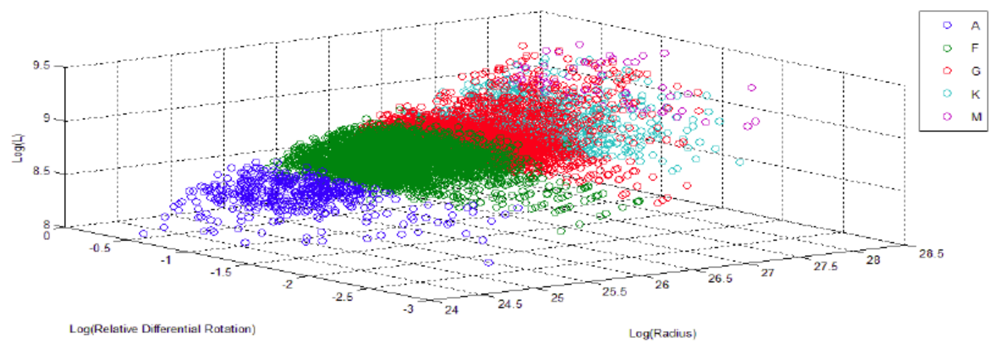

Figure 4: 3D of main sequence (A, F, G, K and M) Kepler's stars (Log RDR vs Log L).

Excluding IL HYa star improved the model restricting the error to only 0.351552. Applying equation 4 to the data of Doppler imaging stars the maximum and minimum differences between original data and forecasted data is given as follows.

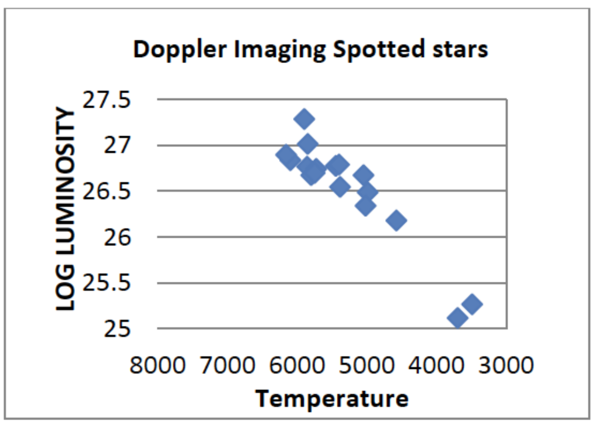

Figure 5: Luminosity and effective temperature relationship of spotted stars mentioned in Table: 1 using the formula $L = 4\pi R^2 \sigma T_{eff}^4$.

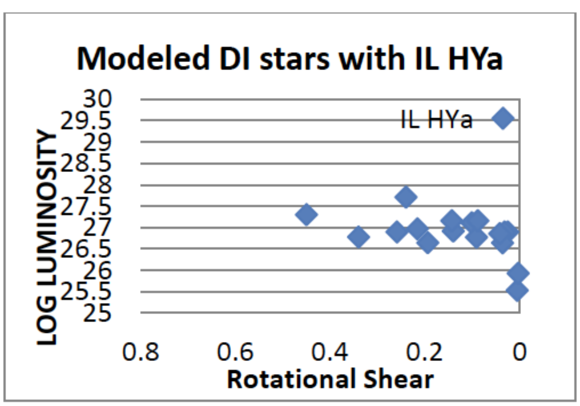

Figure 6: Luminosity and Rotational shear ($\Delta\Omega$) relationship for DI stars using the model equation 4.

$$< L_{star} - L_{model} >=^{\max 0.182}_{\min 0.00000307}$$

Similarly by using equation 8 the difference appears as follows.

$$< L_{star} - L_{model} >=^{\max 0.17587}_{\min 0.091}$$

Figure **7** depicts the log-log model of spotted (DI) stars. Figures **5**, **7** and **8** closely resemble.

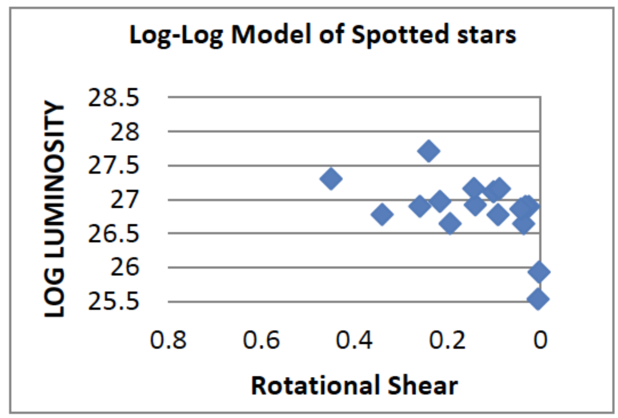

Figure 7: Luminosity and Rotational shear ($\Delta\Omega$) relationship for DI stars modeled by equation 4 excluding IL HYa.

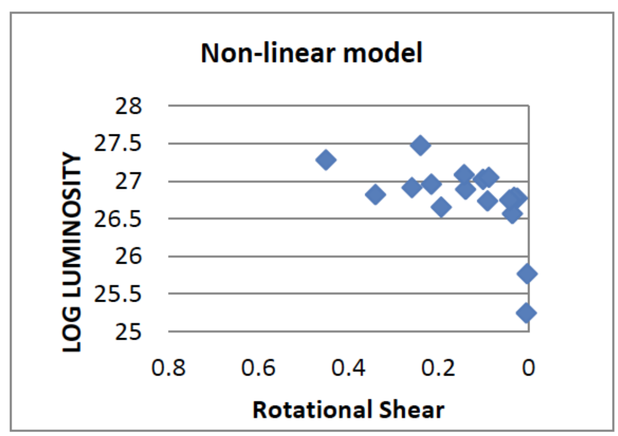

Figure 8: Luminosity and Rotational shear ($\Delta\Omega$) relationship for DI stars modeled by equation 8 excluding IL HYa.

5. DISCUSSION AND CONCLUSION

To establish a relation between stellar spots and stellar evolution no one can ignore the importance of HR diagram which is a footprint of stellar evolution. It is observed that stars, either fully convective or with convective envelope like sun having relative differential rotation α > 0 have spots. They are cool and belong to F-M spectral classes. DI technique uses direct tracking of spotted stars and their DR at different latitude on the surface of stars [20]. Kepler's mission provides a firm helping hand in the study of spotted stars and their DR by photometry [11, 21]. Kepler's data is used to find relations between different evolutionary parameters and to model them. The data was also used to find relation between evolutionary parameters and surface rotations. It is found that rotational shear ΔΩ have strong dependency over effective temperature as indicated by the power law relationship mentioned in section 2. By using this relation with the luminosity and radius of stars multivariate statistical models were forms. To find a possible relationship between Luminosity and surface rotation multivariate linear, log-log multivariate regression and multivariate log-log polynomial (2, 2) degree regression models are constructed. The multivariate log-log polynomial (2, 2) degree regression model 7 and 9 is found to be a best fit model with 87% standard error.

This regression model is also applied to main sequence, F, G, K and M spectral classes of stars. In each case a good relation with R^2 equal to 95%, 96%, 94%, 87% and 96% respectively is found. The results for spectral class "A" of stars are not included in the discussion as it is already confirmed by DI and Photometry techniques that stars with radiative envelope of unstable or undefined convective envelope don't have spots on the surface. The results obtained by applying the log-log model to class "A" stars also confirm the above conclusion.

Relative differential rotation decreases as the star evolves in the main sequence. Constructing the same log-log model (luminosity vs radius and relative differential rotation) it is found that the model is a best fitted model with 89% R^2. The models were further improved by multivariate Log-Log Polynomial (2,2) degree regression equations 7 and 9. The above model shows better results for the main sequence and F-M spectral classes of stars. The corresponding accuracies are depicted in Table **3**. In each case the model is verified by applying to the remaining data obtaining a good accuracy. The model was also tested by eq. 3 for DI spotted stars as mentioned in Table **1**. The standard error is 0.351552. Using the same equation the standard error comes out to be 0.1665 in case of Kepler's data.

Applying the model on DI spotted stars, in view of the appearing errors it is found that the adequacy of the model for DI spotted stars data is weak. In case of DI data the maximum and minimum difference of luminosity calculated by equation 1 luminosity obtained by equation 4 appears as follows.

$$< L_{star} - L_{model} >=^{\max 0.182}_{\min 0.00000307}$$

And by equation 8 is

$$< L_{star} - L_{model} >=^{\max 0.17587}_{\min 0.091}$$

This study forms a basis for further research on the relations between stellar evolution and structure and formation of star spots. The study presented here is the part of the doctoral thesis of first author.

ACKNOWLEDGEMENT

"This research has made use of the NASA Exoplanet Archive, which is operated by the California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program". We are also thankfull to Timo Reinhold and Laurent Gizon (2015) for the data of rotational shear (ΔΩ) and relative differential rotation (α) [22].

REFERENCES

[1] Bowers R, Deeming T, Astrophysics I, Stars, Jones and Bartlett Publishers 1984.

[2] Bray R Loughhead, Sunspots, Chapman and Hall Ltd., London 1964.

[3] Gurevich LE, Lebedinskii AI. Zhur Eksp Teor Fiz 1946; 16: 840.

[4] Romanchuk PR. The formation of sunspots and solar magnetic fields. I, Soviet Astronomy-AJ 1963; 7(3).

[5] Eggenberger P, Meynet G, Maeder A. *et al*. A&A, 2010; 519: A116.

[6] Ceillier T, Eggenberger P, Garcia RA, Mathis S. A&A, 2013; 555: A54.

[7] Marques JP, Goupil MJ, Lebreton Y, *et al*. A&A, 2013; 549: A74.

[8] Skumanich A. ApJ 1972; 171: 565.

[9] Noyes RW, Hartmann LW, Baliunas SL, Duncan DK, Vaughan AH. ApJ 1984; 279: 763.

[10] Soderblom DR, Duncan DK, Johnson DRH. ApJ 1991; 375: 722.

[11] Reinhold T, Reiners A, Basri G. A&A 2013; 560: A4.

[12] Barnes JR, Collier Cameron A, Donati J-F, James DJ, Maesden SC, Petit P. MNRAS 2005a; 357: L1.

[13] DeLeeuw J. information to Akaike (1973) information theory and n extension theory and an extension of the maximum likelihood principal" in Kotz S, Johnson NL. Breakthoughts in Statistics I, Springer 1992; pp. 599-609.

[14] Bierens HJ. Topics in Advance Econometrics: Estimation, testing, and specification of cross-section and time series models, Cambridge university press 1994.

[15] Bowers R, Deeming T. Astrophysics I, Stars, Jones and Bartlett Publishers 1984.

[16] Bray R, Loughhead R. Sunspots, Chapman and Hall Ltd., London 1964.

[17] Stressmeier KG. Doppler images of starspots, Astronomische Nachrichten 2002; 323(3/4): 309-316.

[18] Klaus G. meier, Starspots, A&A Review 2009; 17: 251-308.

[19] Balona LA, Abedigamba OP. Differential rotation in K, G, F and A stars. MNRAS 000 2011; 1-11.

[20] Collier Cameron A, Horne K, Penny A, Leigh C. MNRAS, 2002; 330: 187.

[21] Nielsen M, Gizon L, Schunker H, Karoff C. 2013; arXiv preprint: 1305.5721

[22] Reinhold T, Gizon L. Rotation, differential rotation, and gyrochronology of active Kepler stars. Astronomy & Astrophysics 2015; 583: id.A65, 15: 583. Available at: http://arxiv.org/abs/1507.07757

Received on 28-09-2017 Accepted on 15-11-2017 Published on 02-02-2018

https://doi.org/10.6000/1927-5129.2018.14.01