The Impact of Technology in Mathematics Education: A Case Study of Pakistan

M. Arif Hussain[1], Mujeeb Ahmed Khan[1,2], Shaheen Abbas[2,*], M. Ilyas[2], Bulbul Jan[2] and M. Rashid Kamal Ansari[3]

[1]*Institute of Business and Technology (BIZTEK), Karachi, Pakistan*

[2]*Department of Mathematical Sciences, Federal Urdu University of Arts, Sciences and Technology, Karachi, Pakistan*

[3]*Department of Mathematics, Sir Syed University, Karachi-75300, Pakistan*

Abstract: As we know that computers have become fountain pens of twenty first century. This paper discusses Computer Algebra System (CAS) activities used as part of a study conducted by the authors using *Mathematica*. It also emphasizes the use of *Matlab* for students and researchers for the purpose of verification of results of the problems at hand. This study will attempt to show the positive effect of the use of technology on students' motivation by taking three examples from calculus, advanced algebra and sequence and series problems. The graphic illustration of functions helps students towards a proper understanding of mathematical science problems. The work carried out will be useful for students, researchers, and teachers understand both theory and use of computer applications in order to become master of the subject.

Keywords: Computer Algebra System, *Mathematica* and *Matlab*, mathematical science problem.

1. INTRODUCTION

Computers have become fountain pens of twenty first century. This paper discusses Computer Algebra System (CAS) activities used as part of a study conducted by the authors using *Mathematica* and *Matlab*. It also emphasizes the use of *computer applications* for students and researchers for the purpose of verification of results of the problems at hand. Effective use of technology, like *Matlab* has the potential to stimulate the learning of mathematics, and it is recommended that graduates should be equipped with the skills necessary to exploit such resources [1]. Technology also helps in getting Solution for a class of Lasota–Wazewska model with Infinite delays [2]. Students will be introduced to many topics from contemporary mathematics. These are especially of interest to students in the contemporary educational disciplines. Topics include calculus, advanced algebra, and sequences and series. Computer algebra systems (CAS) are gaining increasing interest in the everyday work of students. Using computer applications helps students a lot in gaining insights of problems. However, students erroneously assume that CAS tools always know more than they do about how to compute a mathematical expression. It is recommended for students to perform calculations in more than one way

in order to counter check final results of the problems at hand [3], as sometimes CAS tools show erroneous results [4].

Research also shows that students learn more from an organized lecture when they are technology-based [5, 6]. Some teachers say that [7, 8] *Mathematica* and Maple are two such systems with which we can create rich learning experience for our students. There has been a wide range of computer activities [9, 10] used in teaching mathematical concepts. In order to enhance students' understanding of abstract linear algebra concepts, the author worked on classroom activities supported by *Matlab* and *Mathematica* to provide inquiry-based learning environments. *Mathematica* also has powerful graphics capability [11]. The communication will attempt to show the positive effect of the use of technology on students' motivation by taking three examples from calculus, advanced algebra and sequence and series problems**.**

Section 2 demonstrates some basic commands of popular computer algebra systems. In Section 3, we explain the use of *Matlab* and *Mathematica* to attack a few mathematical problems. Finally, Section 4 concludes this paper.

2. BASICS OF *MATLAB* AND MATHEMATICA

This Section gives the basics of popular computer algebra systems (CAS), namely *MATLAB* and *Mathematica* for the specification of numerical as well

*Address correspondence to this author at the Department of Mathematical Sciences, Federal Urdu University of Arts, Sciences and Technology, Karachi, Pakistan; E-mail: shaheen838@yahoo.com

as symbolic calculations, and linear programming (LP) commands, which in turn helps in getting solution to a given optimization problem. These tools generally provide partial functionality of the kind to be created in many real world projects. We try to illustrate important features of these tools in LP problem solving. Students are encouraged to develop their own problem solving skills with CAS tools. In addition, all these computer activities are supposed to be augmented by classroom lessons where ideas are discussed and applied.

2.1. Some Basic Operations in *MATLAB*

Addition of numbers in Matlab:

>> 2 + ½

ans =

2.5000

This shows use of MATLAB as a calculator. Now, we look at the matrix representation in two different formats.

>> b=[2,2,3;2,1,9;0,2,3];

>> c=[1 2 3;2 0 9;0 0 3];

>> b'

ans =

2   2   0

2   1   2

3   9

>> c'

ans =

1   2   0

2   0   0

3   9   3

The commas in the specification of **b** can be replaced with spaces, as depicted by matrix **c**. Square brackets refer to vectors and parentheses are used to refer to elements within a matrix. The command **det** and **inv** return the determinant and inverse of a matrix respectively. The ' performs the transpose of a matrix. Transpose of matrices **b** and **c**, have been shown above.

The **help** command returns information on different commands.

2.1.1. Calculus with MATLAB

Differentiation

For constructing symbolic objects, we define

>> syms x
% defines *x* as symbolic variable
>> y=x*x; diff(y,x)
% **diff** differentiates *y* w.r.t. *x*

ans =

2*x

For its second derivative we write

>> y=x*x;diff(y,x,2)

ans =

2

Integration

MATLAB uses **int** command to integrate. For example, int(S, x) is the indefinite integral of S with respect to 'x'. Similarly, int(S, v, a, b) is the definite integral of S with respect to v from a to b. We demonstrate these with few examples.

>> syms x t;
>> int(1/(1+x^2))
ans =
atan(x)

2.2. Symbolic Computations in *Mathematica*

[Input]:   *x* + 3*x*

[Output]:   4*x*

Mathematica has many powerful features which go far beyond even what is needed for calculus. We define some functions for the purpose of evaluation. The syntax for defining function is as follows:

g[x_] := x^4 + 1;

h[x_] := −2*x + 6;

Here, we defined two functions *g*(*x*) and *h*(*x*). It is necessary to use := and the underline after the variable. It is also important to note that functional dependence in *Mathematica* is always denoted by square brackets **[ ]**.

[Input]: **g[2]**

[Output]: 17

Similarly,

[Input]:   **h[2]**

[Output]:   2

2.2.1. Calculus in Mathematica

Differentiation

The derivative $g'(x)$ can be obtained by using command: **D[ ]** as given below.

[Input]:   **D[g[x],x]**

[Output]: $3x^2$

Integration

We define another function $f(x)$:

   f[x_] := 1 / (x + 5);

Now, we perform integration in *Mathematica* environment.

[Input]:   **Integrate[f[x],x]**

[Output]:   Log[x+5]

The above result is exact. One can also obtain numerical integration of the same function as follows.

[Input]:   **NIntegrate[f[x], {x,2,4}]**

[Output]:   0.251314

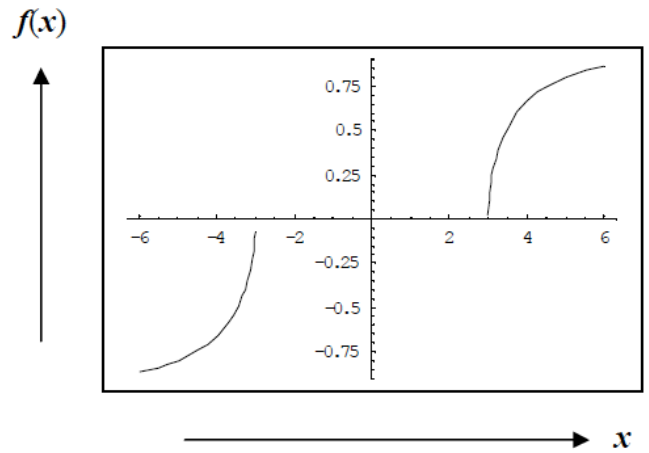Next, we consider the three case studies in section 3.

3. CASE STUDIES

This section considers three case studies
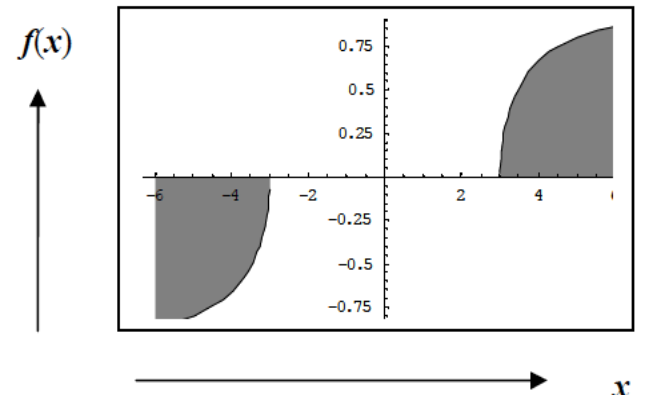
EXAMPLE 1:   Even   and   Odd   function Integration [12]

Plot of odd function: $\int_{-6}^{-3} \dfrac{\sqrt{x*x-9}}{x}\,dx$ is given below. The plot is from -6 to 6 limits. We observe that

Odd function lies in the first and the third quadrant.

Figure 1: Plot of odd function $\dfrac{\sqrt{x*x-9}}{x}$ from –6 to 6 limits.

Plots appear in the first and the third quadrants as the function can not be plotted near zero.

Figure 2: Filled plot of odd function $\dfrac{\sqrt{x*x-9}}{x}$ from –6 to 6 limits.

Input: $\int_{-6}^{-3} \dfrac{\sqrt{x*x-9}}{x}\,dx$

Output: - 3   3 + p = -2.05456

Both *MATHEMATICA* and *MATLAB* applications software give *correct* answers as depicted in Figure **3** (area being in the third quadrant).

Answer in the book is:

$3(\sqrt{3} - \dfrac{\pi}{3}) = 2.05456$

which is incorrect.

Actually the book solution starts with the supposition

$x = 3 \sec\theta, \quad dx = 3 \sec\theta \tan\theta \, d\theta.$

$\Rightarrow \sqrt{x*x - 9} = 3 \tan\theta$

The limits for are calculated as follows:

when $x = -6$, $\qquad \sec\theta = -2$ $\qquad \theta = \dfrac{2\pi}{3}$

when $x = -3$, $\qquad \sec\theta = -1$ $\qquad \theta = \pi$

and all these substitutions give

$\displaystyle\int_{-6}^{-3} \frac{\sqrt{x*x - 9}}{x} \, dx = 3(\sqrt{3} - \frac{\pi}{3}) = 2.05456.$

In fact, one should use the transformed integral as

$-3\displaystyle\int_{-\pi}^{-\frac{2\pi}{3}} \tan\theta^2 \, d\theta = -3\left(\sqrt{3} - \frac{\pi}{3}\right) = -2.05456.$

This is obvious, as solution of $\sec\theta = -2$ gives two solutions $\theta = \dfrac{2\pi}{3}$, and $\theta = -\dfrac{2\pi}{3}$

Similarly, solution of $\sec\theta = -1$ gives two solutions $\theta = \pi$, and $\theta = -\pi$

As the curve is in the third quadrant (both limits appear in the 3rd quadrant), therefore, one should use the above limits with reverse order, that is, $-\pi$ (for lower limit) first then $-\dfrac{2\pi}{3}$ (for upper limit). As the limit orders are revesred, therefore, a minus sign is put before number 3. We can say that plotting of such functions before evaluation of integrals is useful in finding correct limits of integrals.

Next, we consider an example from advanced algebra.

EXAMPLE 2: A problem in Linear Programming (LP) [13]:

1)   Solve the LP problem:

Maximize $F = 2 x_1 + x_2 + 3 x_3$

Subject to: $x_1 + 2x_2 + x_3 \leq 12$

$2 x_1 + x_2 + x_3 \leq 20$

$x_1 + x_2 + 2 x_3 \leq 20$

$x_1, x_2, x_3 \geq 0$

Matlab command **linprog** fails to solve this simple example. The same set of examples will be solved by *Mathematica* in the next section.

Mathematica Solution

We use two different commands of Mathematica to solve above the first command ***LinearProgramming.***

1)   Linear Programming [{-2,-1,-3},{{-1,-2,-1},{-2,-1,-1},{-1,-1,-2}},{-12,-20,-20}]

Output: {**4, 0, 8**}

Maximum ***F*** comes out to be **32**. at $x_1 = 4$,

$x_2 = 0$, and $x_3 = 8$

And now we use another command ***Maximize***

2)   Maximize[{2a+b+3   c,a+2   b+   c≤12,2a+ b+c≤20,a+ b+2 c≤20,a≥0,b≥0,c≥0},{a,b,c}]

Here, a = x_1 b = x_2 , c = x_3 .

Output is: ***F*** = **32**, a→ **4**, b→ **0**, c→ **8**

Above answer is same as given in the reference text. We got correct answer from *Mahematica*.

Therefore, it is recommended that one should be able to use at least two parallel computer applications to counter check results.

Now, we consider an example from infinite series.

How to find the series sum? By using sum of two or three series

It is well known that a sequence $\{S_n\}$ is a function on the set of real numbers, and is usually written as

$s_1 , s_2 , s_3 , \ldots , s_n , \ldots.$

A convergent sequence $\{S_n\}$ is a sequence whose terms will approach a finite value a as $n \to \infty$. We say that $S_n \to a$ as $n \to \infty$.

Symbolically, $\displaystyle\lim_{n\to\infty} S_n =$ L, which is called the limit of the convergent sequence.

Example 3: Approach   to   Convergence   (or Divergence)

Discuss the Convergence (or Divergence) of infinite series: $\dfrac{2}{3} + \dfrac{3}{4} + \dfrac{4}{5} + \dots$

Solution

The nth term of the series is

$$S_n = \frac{n+1}{n+2} = 1 - \frac{1}{n+2}$$

$$\Rightarrow \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(1 - \frac{1}{n+2}\right) = 1 \neq 0.$$

The Divergence theorem does not imply that $\sum S_n$ converges. To confirm whether the series is divergent proceed to find partial sums as follows.

$$S_1 = \frac{2}{3} > \frac{1}{3}$$

$$S_2 = S_1 + \frac{3}{4} > S_1 + \frac{1}{3} > \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$S_3 = S_2 + \frac{4}{5} > S_2 + \frac{1}{3} > \frac{2}{3} + \frac{1}{3} = \frac{3}{3}$$

and so on

$$S_n > S_{n-1} + \frac{1}{3} \quad \forall n > 1 \quad (1)$$

and

$$S_n > \frac{n}{3} \quad \forall n \geq 1 \qquad (2)$$

(1) $\Rightarrow \{ S_n \}$ is monotonically increasing.

(2) $\Rightarrow \{ S_n \}$ has no upper bound as

$n \to \infty$.

That is, $\lim\limits_{n \to \infty} S_n = \infty$. Hence the series is divergent. Such detailed working is necessary for the proofs of mathematical drills. *Mathematica* and *Matlab* are able to give final limits, but these computer applications are to give such details. Therefore, it is recommended for students, researchers, and teachers understand both theory and use of computer applications in order to become master of the subject.

4. CONCLUSION

Three mathematical examples related to calculus, advanced algebra and sequences and series have been explored using computer algebra systems to guide students, researchers and teachers. Reported case studies are the result of ongoing research investigating the effect of technology on teaching and learning. Results revealed that for better understanding of mathematical problem solving, students and teachers should know mathematical theory and the use of appropriate commands in *Matlab* or *Mathematica* to verify the results. In fact, many factors influence the implementation of technology in imparting such know how to students. Finally, we can say that *Matlab* and *Mathematica* can be used to improve the students' understanding of the important results of the problems at hand. Further discussion of such problems will be the topic of a future paper.

REFERENCES

[1]   Cretchley P, Harman C, Ellerton N, Fogarty G. MATLAB in Early Undergraduate Mathematics: An Investigation into the Effects of Scientific Software on Learning. Math Educ Res J 2000; 12(3): 219-33.
http://dx.doi.org/10.1007/BF03217086

[2]   Zhou H, Zhou Z, Wang Q. Positive Almost Periodic Solution for a class of Lasota–Wazewska Model with Infinite Delays. Appl Math Compu 2011; 218(8): 4501-506.
http://dx.doi.org/10.1016/j.amc.2011.10.031

[3]   Aguirregabiria JM, Hernandez A, Rivas M. Are we careful enough when using algebra system. Comput Phys 1994; 8(1).
http://dx.doi.org/10.1063/1.4823260

[4]   Mortberg A. Constructive Algebra in Type Theory, Thesis for the Degree of Licentiate of Engineering, Department of Computer Science and Engineering Chalmers University of Technology and University of Gothenburg Gothenburg, Sweden 2012.

[5]   Hoyles C, Noss R. What can digital technologies take from and bring to research in mathematics education? In: Bishop AJ, Clements MA, Keitel C, Kilpatrick J, Leung F, Eds. Second International Handbook of Mathematics Education. Dordrecht: Kluwer Academic Publishers 2003; Vol. 1: pp. 323-349.
http://dx.doi.org/10.1007/978-94-010-0273-8_11

[6]   Kendal M, Stacey K. Varieties of teacher privileging for teaching calculus with computer algebra systems. Int J Comput Algebra Math Educ 1999; 6: 233-47.

[7]   Wicks RJ. *Mathematica* materials from MAA mini-course on creating interactive texts in *Mathematica*. MAA 1999 Joint meetings.

[8]   Wicks RJ. Linear Algebra: an interactive laboratory approach with *Mathematica.* Addison-Wesley Publishing Company, Reading, Massachusetts 1996.

[9]   Shaw WT, Tigg J. Applied *Mathematica*: Getting Started, Getting It Done. Addison-Wesley, Massachusetts 1994.

[10]   Davis B, Porta H, Uhl J. Calculus andd *Mathematica*:, Addison-Wesley, Massachusetts 1994.

[11]   Maeder R. Programming in *Mathematica.* Addison-Wesley, Massachusetts 1999.

[12]   Qureshi MAR, *et al*. Mathematics for Class XII, Gabba Educational Books, Karachi 2009; p. 233.

[13]   Hussain MA, Rahim I. Teaching Optimization Techniques using MATLAB and MATHEMATICA: A Comparative Study. J Inform Commun Technol 2012; 6(1): 18-26.