Efficacy of Non-Linear Approach in the Study of Ozone Layer Depletion

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Abstract: The stratosphere is one of the constituents of thermal structure of the atmosphere. The maximum concentration of ozone is found at the stratospheric region where it is interacted by many species including chemical and physical processes. Atmosphere as a whole is an open system that is regarded as a non linear system and that seems to be complex. Therefore, a non-linear trend is plausible to explain phenomenon of ozone layer depletion (OLD).

In this manuscript we have paid our attention in the analysis of the major portion of historic data on stratospheric O_3 based on ground-based measurements by the Dobson Spectrophotometer. In this communication we have estimated parameters for describing non-linearity in the process using polynomial trend functions and predicted values are calculated for the period from 1960 to 1999. Future values for ozone depths are computed till 2006 and compared with the minor portion of the data set.

Keywords: Ozone layer depletion, Non-linear approach, Pakistan Air Space, Bilinear model (BARMA), Stratospheric Ozone.

1. INTRODUCTION

The field of atmospheric physics is treated in general with deep suspicion because of the complexity of the processes and variability in the concentration of the atmospheric constituents. In this communication we intend to study the influence of ozone (O_3) column in the stratospheric region of Pakistan. Quite a few reported findings have survived the subsequent checks. The atmosphere is the recipient of many of the products of our technological society. These effluents include products of combustion of fossil fuels and of the development of new synthetic chemicals. Classical examples include the realization in the 1950s that motor vehicle emissions could lead to urban smog and the realization in the 1970s that emissions of the chlorofluorocarbons from the aerosol spray cans and refrigerators could cause the depletion of stratospheric O_3 [1-3].

It follows from the theory of ozone destruction that the man-made activities may constitute a threat to stratospheric O_3. This possibility was first stressed by stating that the supersonic aircraft flying in the stratosphere could emit nitrogen oxides in a quantity to alter chemical processes. The total ozone and the vertical profile of ozone were measured at several stations using Dobson spectrophotometers during 1920s and 1930s. As it is known that the circulation pattern is strong in winter and spring when stratospheric wind moves downward over polar regions and upward over the tropics [4-6].

The class of models considered by Box-Jenkins has been used with considerable success in many scientific fields. It is natural to ask if there exists a more general class of models that will bestow a better fit to the reality. Recent obvious directions to put one step forward is to implement non-linear approach [7, 8].

The phenomenon of ozone layer depletion (OLD) is complex because ozone is situated in the stratosphere which is an open system where the ozone is interacted by many species including chemical and physical processes. Therefore, a non-linear trend is plausible to explain this intricacy of the phenomenon [9, 10].

There is very important area in the mathematical field to which we have paid our attention is the analysis of the major portion of historic data on stratospheric O_3 based on ground-based measurements by the Dobson Spectrophotometer. The same instrument has been installed at Quetta, Pakistan under the auspices of

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ISSN: 1814-8085 / E-ISSN: 1927-5129/13

WMO. We obtained ground-based data of ozone depth from this center. In assessing the effect of OLD on the biosphere, mathematization will be essential and fewer systems are more complex to predict their behaviour. In fact, the atmosphere is intricate and energy interacted between the earth's surface, solar radiation, and the atmosphere are complicated, we have employed experimental data and incorporate the laws of physics [11-13]. In section 2, we mentioned methodology for estimating parameter of non-linear model. In section 3, we have determined the size of the OLD by computing the mean and the standard deviation of the data space. In section 4, a linear model has been constructed to show the actual situation of OLD at Pakistan's atmospheric regions. In section 5, non-linear behaviour of the OLD determined using polynomial trend functions and predicted values are calculated for the period from 1960 to 1999. Future values for ozone depths are computed till 2010 and compared with the minor portion of the data set. In section 6, nonlinear estimation is done using bilinear model. The parameter values are calculated and substituted in the model equation to get predicted value of the depth of ozone. Finally in section 7, we concluded the paper.

Calculated mean & SD defining the sample data.

2. METHODOLOGY

In this communication we have utilised first-order bilinear model also known as BARMA. Bilinear models were studied in the context of non-linear control, but their application as time series models have been investigated [14, 15].

The analysis of single time series has been used for a long time in the predictions of different atmospheric events. This use in providing forecasts for comparing with alternative techniques is justified enough with great concern in many scientific fields including atmospheric sciences. A suitable class of models does exist that could manifest better fits to reality, obviously by considering time varying parameters. Some of the special classes of non-linear models which have attracted the attention of the researchers are mentioned [5, 16]. In this study we intend to forecast the behaviour of OLD using such kinds of non-linear models. We have utilized the bilinear model in order to forecast OLD.

This study appraises results in terms of physical interpretation and the methodology used that could be helpful in obtaining predictions of use to various organisations in the private, the public, and the government sectors.

3. SIZE OF STRATOSPHERIC RESIDENT O_3 FLUCTUATIONS

Of available standard classical and modern tools for parameter estimation for our current context we may choose the Jackknife estimator technique because of its several useful properties discussed in the standard literature. The mean and standard deviation are calculated using the Jackknife technique as

$\bar{X}$ = 283.65, and σ = 19.68

and compared with the values of these parameters obtained by the MLE technique

$\bar{X}$ = 283.37, and σ = 19.65

4. PRELIMINARY REGRESSION MODELING

We have computed and graphed the secular trend using simple regression approach and our result depicted in Figure **1**, established that the O_3 layer depletion is a threat to mankind and it is also affecting Pakistan's environment.

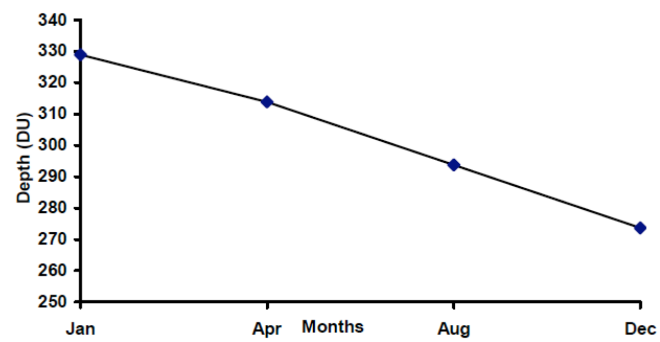

Figure 1: Secular trend by least square method for the year 1982.

This linear model seems to have practical implication and helpful making predictions for various national and international organizations. These and some of their variants will be tackled in future [1].

5. NON-LINEAR BEHAVIOUR

Non-linear character comprises the following:

(1) Quadratic or cubical shape of the OLD data using polynomial trend functions

(2) test of removing higher orders of the terms in the polynomial trend curves

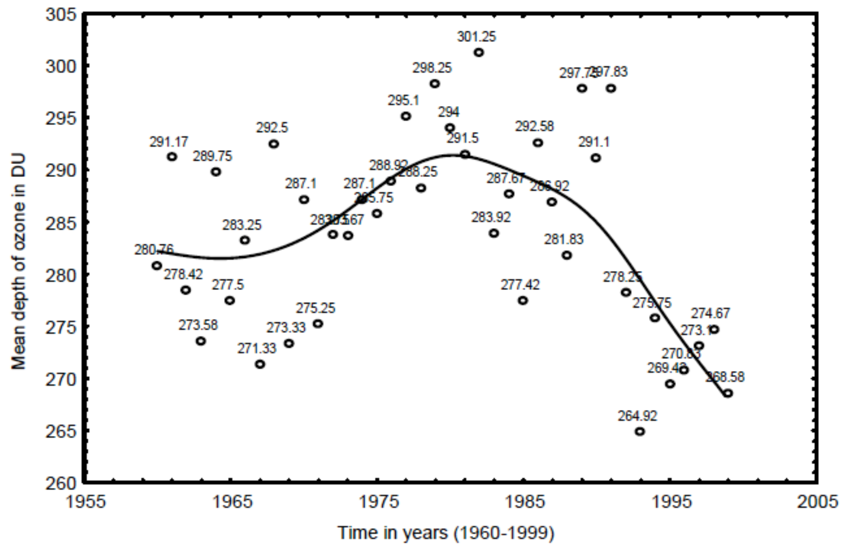

Figure 2: Illustrations of mean depth of Pakistan's stratospheric ozone columns for the period 1960 – 1999.

(3) Contribution of the respective terms.

(4) Finding estimators

Using the strategies (1-4) When graphed (cf Figure **2**) overtime many atmospheric phenomena approximately follow a quadratic or cubic or higher order curves rather than a straight line In such cases it is appropriate to fit a quadratic or cubic curve. Occasionally a cubic or higher order curve is suitable to describe the OLD over time.

The general polynomial trend model is defined as follows;

$$y_t = \beta_o + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \ldots + \beta_s t^s + \varepsilon_t \qquad (1)$$

y_t = Value of dependent variable (O_3depth) at time t u

t = the unit time

s = degree of the polynomial

ε_t= Random error term im time t.

The principle is straight forward and the mathematical details are examined. If we are using the depth of the O_3 layer as dependent variable and time in years as the independent variable then we could obtain a curve. We obtain a non-linear trend by weighted least square for O_3 layer using a model equation

$$y_t = \beta_o + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \varepsilon_t \qquad (2)$$

What this says is observed values are modelled in terms of a pattern and a random error term

$$y_t = \text{(pattern)} + \text{(error)} = \hat{y} + \text{(error)} \qquad (3)$$

n = 40,

β_o = 285.113

β_1 = 1.097

β_2 = 0.018

β_3 = 0.015

$\hat{y}$ = 285.113-1.097*x-0.018*x^2+0.015*x^3

R^2 = 0.792 as Coefficient of determination for this model.

x- is considered as time (t)

This equation can predict future value of the ozone depth by

(i) x =6, 281.12 DU

(ii) x =7, 280.03 DU, year 2006

Further inserting x =10 that is for time t=10

$\hat{y}$ = 285.113-1.097*10-0.018*10^2+0.015*10^3 = 287.34 DU

For x = 11, year 2011,

$\hat{y}$ = 285.113-1.097*7-0.018*7^2+0.015*7^3 = 290.81 DU

Notice that this estimation of the ozone layer depletion for Pakistan's atmospheric region is due to the assumption that the past trends values will continue unchanged in the future. However, if the atmospheric conditions change, then another predicting such as bilinear AR technique will be essential.

6. ESTIMATION OF MODEL PARAMETERS

The bilinear modeling was used to estimate the parametric values of OLD. We know that this modeling resembles the general bilinear autoregressive moving average model of order (p,q,Q,P), BARMA (p,q,Q,P), that is of the form

$$X_t = \sum_{j=1}^{P} a_j X_{t-j} + \sum_{i=0}^{q} b_i \varepsilon_{t-i} + \sum_{k=0}^{Q} \sum_{\ell=1}^{P} c_{k\ell}\, \varepsilon_{t-k} X_{t-\ell}, \quad (4)$$

where $b_0 = 1$

The advantage of ARMA model is that fitting the above equation would, in general require far fewer parameters than fitting a pure AR and MA model. Bilinear models were studied in the context of non-linear control, but their application as time series models has been investigated The bilinear class, may be regarded as the natural non-linear extension of an ARMA model [15, 16]. The first-order bilinear model also known as BARMA is given by

$$X_t = aX_{t-1} + b\, Z_t + c\, Z_{t-1} X_{t-1} \quad (5)$$

where $\{Z_t\}$indicates a purely random process and a, b, and c are parameters. The last term on the right hand side of the above equation is non-linear term.

$$X_t = aX_{t-1} + b\, Z_t + c\, Z_{t-1} X_{t-1} \quad (6)$$

$$V_1 = A*V2 + B*Rnd + C*\ Rnd2*V2 \quad (7)$$

# of parameters to be estimated: 3

Loss function: (Obs – Pred)**2

# of iterations: 12

Parameter estimate process is converged, R = 0.542, R^2 = 0.297

Proportion of variance account for = 0.291

Estimation method: Qusai Newton

Max.# iterations: 50.

Convergence Criterion: 0.0001

Parameter estimation Process converged: # of iteration:12

Parameters	A	B	C
Estimate	0.998	5.295	–0.000034
Std. Error	0.003	3.733	0.000591
t (476)	374.151	1.418	–0.05730
p - level	0.000	0.156	0.954327

Using equation (7), the value of the depth of for 481st month can be calculated as

X_{481} = 261.29 DU (for data I)

X_{297} = 258.48 DU (for data II)

Data point 481for the month of January in 2000

Data point 297 for September 1984.

Probability Distribution Parameters

Variables	Mean	Standard deviation
V1	283.65	19.68
V2	283.65	19.65
Rnd	0.0040	0.2033
Rnd2	0.2123	4.5735

Correlation Matrix of Estimates of the Bilinear Model

$$\begin{bmatrix} 1.0000 & -0.0215 & -0.0466 \\ -0.0215 & 1.0000 & 0.0334 \\ -0.0466 & 0.0334 & 1.0000 \end{bmatrix}$$

7. CONCLUSION

We have implemented quadratic, cubic and bilinear models to determine the forecasts by analyzing the time series. These techniques concentrate on interpretation of the non-linear system like OLD. To improve the forecast we have utilized bilinear model.

ACKNOWLEDGEMENT

I would like to thank the Dean Faculty of Science for its support to complete this work. The technical staff of Geophysical Centre has been greatly acknowledged.

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Received on 08-10-2013 Accepted on 22-10-2013 Published on 21-11-2013

http://dx.doi.org/10.6000/1927-5129.2013.09.75