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Catalog of Coefficients for Estimating Bulk and Shear Moduli as a Function of Lithology

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Abstract:

The purpose of this paper is to present correlation coefficients for a variety of rock types that can be used in a suitable petroelastic model (PEM). The correlation coefficients for different rock types facilitate the application of a petroelastic model in reservoir flow models. By combining the correlation coefficients and the PEM, it is possible to obtain low-cost estimates of reservoir geophysical attributes. The rock types include dolomite, limestone, high porosity sandstone, poorly consolidated sandstone, tight gas sandstone, and well consolidated Gulf Coast sandstone.

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INTRODUCTION

A petroelastic model (PEM) can be used to estimate reservoir geophysical attributes that are useful for petroleum engineering and carbon dioxide sequestration calculations. For example, Souza, *et al*. [1] presented a methodology to classify fluid flow models by combining 4D seismic amplitude attributes and reservoir production data. Curcio and Macias [2] combined pressure and saturation distributions from a fluid flow simulator with breakdown criteria and petrophysical relations to simulate fracture propagation and electromagnetic response. Commer, *et al*. [3] attempted to clarify hydrogeophysical parameter estimation concepts when applied to 4D seismic monitoring of fluid injection process. Kalam, *et al*. [4] reviewed several geological sequestration projects.

Reservoir geophysical attributes that can be calculated from the PEM presented in this paper include bulk and shear moduli, compressional velocity and shear velocity, acoustic impedances, dynamic Young's modulus, and Poisson's ratio. Moduli are represented as functions of porosity, effective pressure, and clay content volume fraction. By combining correlation coefficients for the moduli and the PEM, it is possible to represent several existing P-wave velocity and S-wave velocity models, as well as obtain low-cost estimates of other reservoir geophysical attributes.

The purpose of this paper is to present correlation coefficients for a variety of rock types that can be used in a readily accessible PEM [5]. The rock types include dolomite, limestone, high porosity sandstone, poorly consolidated sandstone, tight gas sandstone, and well consolidated Gulf Coast sandstone. We begin by introducing the PEM, and then present correlation coefficients for bulk modulus and shear modulus in different rock types. Flow model applications of the PEM discussed here are presented in Fanchi [5-7] and Almudh'hi and Fanchi [8].

2. THE PETROELASTIC MODEL (PEM)

The PEM is designed to calculate seismic compressional velocity and shear velocity. These velocities are expressed in the functional form:

$$V_p = \sqrt{\frac{K^* + \frac{4\mu^*}{3}}{\rho^*}} \qquad (1)$$

and

$$V_S = \sqrt{\frac{\mu^*}{\rho^*}} \qquad (2)$$

where the variables in a consistent set of units are

V_P	=	compressional velocity
V_S	=	shear velocity
K^*	=	IFM bulk modulus
μ^*	=	IFM shear modulus
ρ^*	=	IFM bulk density = $(1-\phi)\rho_m + \phi\rho_f$
ρ_m	=	the density of rock matrix grains
ρ_f	=	fluid density = $\rho_o S_o + \rho_w \boldsymbol{S_w} + \rho_g S_g$
ϕ	=	porosity

The general PEM represents moduli as functions of porosity ϕ, effective pressure P_e, and clay content volume fraction C. Effective pressure is the difference between confining (overburden) pressure and pore pressure$P$

$$P_{eff} = P_{con} - \alpha P \qquad (3)$$

with correction factor α. Confining pressure P_{con} may be estimated from an average overburden gradient γ_{OB} so that $P_{con} = \gamma_{OB} z$ where z is depth.

The IFM bulk modulus has the form:

$$K^* = K_{IFM} + \frac{\left[1 - \frac{K_{IFM}}{K_m}\right]^2}{\frac{\phi}{K_f} + \frac{1-\phi}{K_m} - \frac{K_{IFM}}{K_m^2}} \qquad (4)$$

where

K_{IFM}	=	IFM dry frame bulk modulus
K_m	=	the bulk modulus of rock matrix grains
K_f	=	the bulk modulus of fluid = $1/c_f$
c_f	=	fluid compressibility = $c_o S_o + c_w \boldsymbol{S_w} + c_g S_g$

The IFM dry frame bulk modulus has the functional dependence

$$K_{IFM} = a_0 + a_1 P_e^{e_1} + a_2\phi + a_3\phi^2 + a_4\phi P_e^{e_2} + a_5\sqrt{C} \qquad (5)$$

with model coefficients $(a_0, a_1, a_2, a_3, a_4, a_5, e_1, e_2)$. Sets of correlation coefficients are presented in Sections 3 and 4. Rock matrix grain modulus K_m is calculated from IFM dry frame bulk modulus K_{IFM} when porosity equals zero, thus

$$K_m = a_0 + a_1 P_e^{e_1} + a_5\sqrt{C} \qquad (6)$$

The functional dependence of shear modulus is

$$\mu^* = \alpha_0 + \alpha_1 P_e^{\varepsilon_1} + \alpha_2\phi + \alpha_3\phi^2 + \alpha_4\phi P_e^{\varepsilon_2} + \alpha_5\sqrt{C} \qquad (7)$$

with model coefficients $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \varepsilon_1, \varepsilon_2)$. Sets of correlation coefficients are presented in Sections 3 and 4.

Rock matrix grain density ρ_m may be expressed as the following quadratic function of clay content

$$\rho_m = b_0 + b_1 C + b_2 C^2 \quad (8)$$

with regression coefficients (b_0, b_1, b_2). The form of Equation (8) lets density be specified as a function of clay volume fraction.

Constant Moduli (Gassmann) Model

Bulk modulus is calculated from Gassmann's equation as follows [9]:

$$K^* = K_{sat} = K_{\text{dry}} + \frac{\left[1 - \frac{K_{\text{dry}}}{K_m}\right]^2}{\frac{\phi}{K_f} + \frac{1-\phi}{K_m} - \frac{K_{\text{dry}}}{K_m^2}}, \quad \mu^* = \mu, \quad \rho^* = \rho_B \quad (9)$$

where

K_{sat}	=	saturated bulk modulus
K_{dry}	=	dry frame bulk modulus
K_m	=	the bulk modulus of rock matrix grains
K_f	=	the bulk modulus of fluid = $1/c_f$
μ	=	shear modulus
ρ_B	=	Bulk density = $(1-\phi)\rho_m + \phi\rho_f$

The dry frame bulk modulus, the bulk modulus of the rock matrix grains, and the shear modulus in the Gassmann model do not depend on effective pressure or clay content.

Dynamic Poisson's Ratio and Dynamic Young's Modulus

Dynamic Poisson's ratio ν is calculated as:

$$\nu = \frac{0.5V_P^2 - V_S^2}{V_P^2 - V_S^2} \quad (10)$$

Dynamic Young's modulus E is calculated from Poisson's ratio ν as:

$$E = 2(1+\nu)\mu \quad (11)$$

where μ is shear modulus.

3. CORRELATION COEFFICIENTS FOR BULK MODULUS AND SHEAR MODULUS IN CARBONATES

The IFM petroelastic algorithm can be used to represent P-wave velocity and S-wave velocity models in carbonates. Coefficients for dolomite and limestone models are presented below.

Dolomite Moduli – GYJ Model

The GYJ model is based on the work by Geertsma, Yale, and Jamieson as presented in Appendix 10.1 of Mavko, *et al*. [10]. The regression model coefficients are presented below.

Limestone Moduli – CLYJ Model

The CLYJ model is based on the work by Cadoret, Lucet, Yale, and Jamieson as presented in Appendix 10.1 of Mavko, *et al*. [10]. The regression model coefficients are presented below.

4. CORRELATION COEFFICIENTS FOR BULK MODULUS AND SHEAR MODULUS IN SANDSTONES

The IFM petroelastic algorithm can be used to represent P-wave velocity and S-wave velocity models in sandstones. Coefficients for different sandstone rock types are presented below.

Table 1: GYJ Model Coefficients for Dolomite Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	1.0579×10^7	α_0	5.1186×10^6
a_1	0	α_1	0
a_2	-3.9834×10^7	α_2	-1.5622×10^7
a_3	3.8525×10^6	α_3	1.3043×10^7
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Table 2: CLYJ Model Coefficients for Limestone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	7.3708 × 10^6	α_0	3.5783 × 10^6
a_1	0	α_1	0
a_2	-2.2735 × 10^7	α_2	-1.0634 × 10^7
a_3	1.8444 × 10^6	α_3	8.6055 × 10^6
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Blangy Model: Poorly Consolidated Sandstone Moduli

The Blangy model is based on the work by Blangy as presented in Appendix 10.1 of Mavko, *et al*. [10]. The regression model coefficients are presented below.

CBE Model: Clastic Silicate Rock Moduli

The CBE model is based on the work by Castagna, Batzle and Eastwood [11]. The regression model coefficients are presented below.

Han Model: Sandstone Moduli

The Han model is based on the work by Han as presented in Appendix 10.1 of Mavko, *et al*. [10]. The regression model coefficients are presented below.

HEP Model: Well Consolidated Gulf Coast Sandstone Moduli

The HEP model is based on the work by Han-Eberhart-Phillips as presented in Section 7.5 of Mavko, *et al*. [10]. The regression model coefficients are presented below [12].

Table 3: Blangy Model Coefficients for Poorly Consolidated Sandstone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	1.4984 × 10^6	α_0	2.8408 × 10^6
a_1	0	α_1	0
a_2	-3.9073 × 10^6	α_2	-6.5569 × 10^6
a_3	2.7870 × 10^6	α_3	3.8515 × 10^6
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Table 4: CBE Model Coefficients for Clastic Silicate Rock Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	5.8523 × 10^6	α_0	5.2611 × 10^6
a_1	0	α_1	0
a_2	-2.4202 × 10^7	α_2	-1.6956 × 10^7
a_3	2.6566 × 10^7	α_3	1.4615 × 10^7
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Table 5: Han Model Coefficients for Sandstone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	4.6235 × 10^6	α_0	3.2642 × 10^6
a_1	0	α_1	0
a_2	-1.2609 × 10^7	α_2	-8.8208 × 10^6
a_3	9.1257 × 10^6	α_3	6.5817 × 10^6
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Table 6: HEP Model Coefficients for Well Consolidated Gulf Coast Sandstone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	5.2001 × 10^6	α_0	4.2958 × 10^6
a_1	2.9300 × 10^4	α_1	5.3952 × 10^4
a_2	-1.4307 × 10^7	α_2	-1.4952 × 10^7
a_3	6.9014 × 10^6	α_3	1.3948 × 10^7
a_4	5.7684 × 10^2	α_4	-2.2544 × 10^4
a_5	-1.1936 × 10^6	α_5	-2.6009 × 10^6
e_1	1/3	ε_1	1/3
e_2	1/3	ε_2	1/3

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Jizba Model: Tight Gas Moduli

The Jizba model is based on the work by Jizba as presented in Appendix 10.1 of Mavko, *et al*. [10]. The regression model coefficients are presented below.

MRH MODEL: QUARTZ SANDSTONE MODULI

The MRH model is based on the work by Murphy, Reischer, and Hsu [13]. The regression model coefficients are presented below.

Strandenes Model: High Porosity Sandstone Moduli

The Strandenes model is based on the work by Strandenes as presented in Appendix 10.1 of Mavko, *et al*. [10]. The regression model coefficients are presented below.

5. CONCLUDING REMARKS

Several sets of PEM correlation coefficients for different rock types are presented in Sections 3 and 4.

Table 7: Jizba Model Coefficients for Tight Gas Sandstone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	3.5426 × 10^6	α_0	3.3554 × 10^6
a_1	0	α_1	0
a_2	-9.0000 × 10^6	α_2	-4.9500 × 10^6
a_3	9.000 × 10^6	α_3	6.5000 × 10^6
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Table 8: MRH Model Coefficients for Quartz Sandstone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	5.2200 × 10^6	α_0	5.9450 × 10^6
a_1	0	α_1	0
a_2	-1.3050 × 10^7	α_2	-1.4863 × 10^7
a_3	0	α_3	0
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

Table 9: Strandenes Model Coefficients for High Porosity Sandstone Moduli*

K_{dry} Coefficient	Regression Value	μ^* Coefficient	Regression Value
a_0	3.5674 × 10^6	α_0	2.1466 × 10^6
a_1	0	α_1	0
a_2	-4.0557 × 10^6	α_2	-3.2338 × 10^6
a_3	1.2744 × 10^6	α_3	1.3933 × 10^6
a_4	0	α_4	0
a_5	0	α_5	0
e_1	0	ε_1	0
e_2	0	ε_2	0

* For K_{dry}, μ^* and P_e in psia; ϕ a fraction; and C a volume fraction. Calculated moduli have units of psia.

The PEM can be used to calculate bulk and shear moduli, P-wave and S-wave velocities, Poisson's ratio and Young's modulus. This paper provides a catalog of correlation coefficients that can facilitate the application of a PEM in reservoir flow modeling.

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