Developing a Mathematical Model to Assess the Liveablity in Blighted Mega City

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Abstract: Karachi (24° 37.38′ N, 66° 54.42′ E) is one of the mega cities of Pakistan. In general, deteriorating urban air quality in developing countries is a worsening environmental problem and causing damage to human health. The urban atmospheric pollution is several times higher than the limits set by the WHO. Being industrial city, the rate of increase of traffic volume has been exponential during the last decade in Karachi. More or less, 90% atmospheric pollution is related to vehicle emissions. To gauge and forecast future traffic volume and urban pollution level mathematical models are needed. This communication attempts to model the urban traffic population evolution and the atmospheric pollution levels during the past 25 years. The traffic model shows that total traffic volume in Karachi was 1 million in 1999. In 2008 it reached 2 million and in 2012 the traffic volume crossed 3 million verifying published data. According to this forecast model, it is also important to note that the total traffic volume in Karachi will go to 4 million, 5 million, and 6 million in the years 2015, 2018, and 2020 respectively. Auto Regressive Integrated Moving Average, ARIMA (2, 1, 2) model is found to be adequate model to capture Karachi urban atmospheric pollution variation. The model is the first of its kind for the region considered. As a further application, we develop an empirical model of local atmospheric pollution fluctuations as determined by urban traffic volume. The work should provide a basis for other applications, including urban planning, urban-regional air quality management, design of efficient energy programs, etc.

Keywords: Urban Traffic Population(UTP), Atmospheric Pollution, Mathematical Model, ARIMA Model, Regression Model.

1. INTRODUCTION

Persistency analysis of cyclone data in Arabian Sea shows that frequency of cyclones formation in the Sea has been rising due to global warming [1-3]. Also, the analysis of sea-surface temperature reveals a positive trend [4]. Moreover, as a result of global climate change, the local wind speed is also increasing [5] in addition to increase in local urban extreme temperature [6]. Generally, the industries and power plants are believed to be major sources of green house gases globally, but in urban areas their contribution to atmospheric pollution is low as compared to road transport. The vehicle population would increase from 60% to 120% by 2025 and 140% to 600% by 2100. In particular, the annual percentage growth in traffic volume is highest in Southeast Asia, Africa, Latin America, and some central and eastern European countries. As a consequence, in megacities all over the world nearly 60% of atmospheric pollution is attributed to road transport [7]. So, the deteriorating urban air quality in developing countries is a worsening environmental problem and causing damage to human health. The urban atmospheric pollution is several times higher than the limits set by the WHO. In case of Karachi, one of the mega cities of Pakistan, and being industrial city, the rate of increase of traffic volume had been exponential during the last decade. In this city, more or less, 90% atmospheric pollution is related to vehicle emissions [8]. The rapid urban population growth has exacerbated the city's planning efforts, and the use of voluminous number of car and motorcycle has outstripped roadway capacities. As a result, congestion problem has been increasing day by day resulting in extra air pollution in the city. There have been several efforts to analyze Karachi urban pollution levels [9-12]. The problem of modeling urban traffic-related atmospheric pollution concentrations has become an important area of inquiry. This communication attempts to model the urban traffic population evolution and the atmospheric pollution (CO_2) levels during the past 25 years. It also attempts to develop a statistically significant regression model of local atmospheric CO_2 fluctuations as determined by total urban traffic volume.

In Section 2, we discuss traffic volume data and traffic-related gas concentration data. Then, in Section 3, we develop the traffic volume evolution model and we do time series analysis and regression analysis

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between pollution concentration and urban traffic volume. Finally, Section 4 concludes the paper.

2. MATERIAL AND METHODS

The impact of human activities on the climate during the Karachi urban development process has been significant. The process of urbanization of Karachi city has undergone different stages, some very peaceful and calm and some haphazard and abrupt [13]. Data set related to traffic volume was obtained from traffic engineering bureau, Civic Centre, Karachi, and from reports prepared by NED Engineering university students [14]. CO_2 atmospheric pollution data was obtained from the published paper [15]. Nonlinear model is fitted to observed data of traffic volume. ARIMA modeling is done for urban atmospheric pollution (CO_2). To verify and bolster the forecast value of urban atmospheric pollution from ARIMA model, a regression model is also developed.

3. TRAFFIC POPULATION AND URBANATMOSPH-ERIC POLLUTION MODELING

This section deals with the traffic volume modeling aspects of our study. Being the nation's largest city with financial, commercial and manufacturing capital, Karachi is the hub of transportation. Total length of road network in Karachi is more than 9,500 km which accommodates about 1.81 million vehicles. As a result the travel speed in most parts of the city is 30 to 40 km/h. Peak travel speeds in the core areas can be 15 km/h or even lower, while daily traffic volumes on major arteries are generally 70,000 to about 180,000 vehicles. The planning and development work needs more ecological and environmentally sustainable development [16]. The following model suggests that the traffic volume in Karachi will reach 6 million in year 2020 and will result in chaotic situation with respect to traffic control in the urban Karachi region. In addition, it will also produce alarming traffic congestion at the intersections, especially at peak hours. Due to large congestion periods, the amount of traffic-related emissions will also enhance.

$$TP(t) = 972080.00 + 49007.42\, t + 11278.00\, t^2 \qquad (1)$$

Here, t =1 means 1997, t = 24 means 2020.

Model (1) also revealed that total traffic volume in Karachi was I million in 1999. In 2008 it reached 2 million and in 2012 the traffic volume crossed 3 million [17]. According to this forecast model, it is important to note that the total traffic volume in Karachi will go to 4 million, 5 million, and 6 million in the years 2015, 2018, and 2020 respectively. The situation is alarming and the urban planners will have to take immediate action in order to cope with the challenge. Projections for the future population are based on current trends and expert guesses about changes that are likely to occur. Even the best projections can be overwhelmed by unexpected events. The scatter plot and fitted model to scatter plot are shown in Figure **1**.

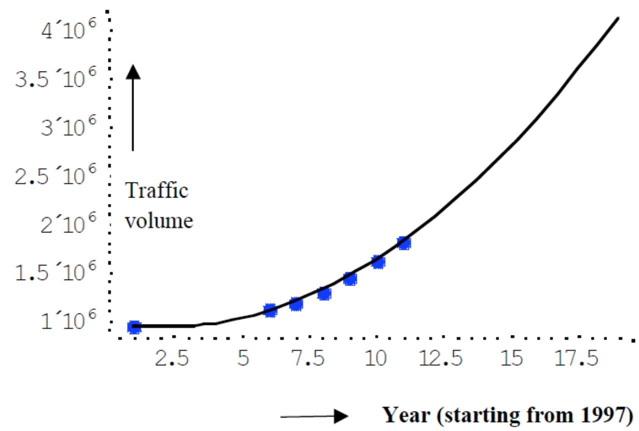

Figure 1: Traffic volume data with fitted nonlinear model to show future values.

3.1. Urban Atmospheric Pollution Model

The most frequently used approaches to time series model building assume that the data under study are generated from a linear Gaussian stochastic process [18]. It is well known that real-life systems are usually nonlinear. Over recent years, several nonlinear time series models have been proposed both in classical econometrics [19-20] and in machine learning theory [21]. It is well known that the primary goals of conducting a time series analysis are:

(a) Characterizing the nature of the phenomenon represented by the sequence of observations, and

(b) forecasting or predicting future values of the time series variable. The focus of this study is on the forecasting capabilities of the models.

The autoregressive, integrated, moving-average (ARIMA) model developed by Box and Jenkins (1976), is denoted as ARIMA (p, d, q) x (P, D, Q)s, where

p = order of the non-seasonal autoregressive process,

d = number of consecutive differencing,

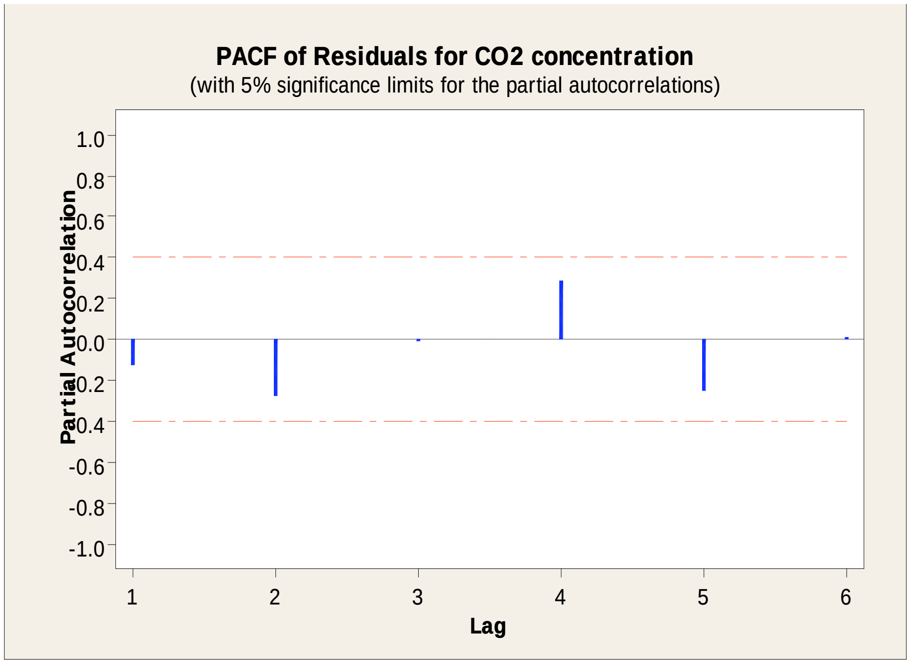

Figure 2: Partial Autocorrelation plot of residuals for ARIMA(2, 1, 2) model.

q = order of the non-seasonal moving average process,

P = order of the seasonal autoregressive process,

D = number of seasonal differencing,

Q = order of the seasonal moving average process, and

s = the span of the seasonality.

The ARIMA methodology has gained enormous popularity in many areas, and research practice confirms its power and flexibility, especially when patterns of the data were unclear and individual observations involved considerable error. The Box-Jenkins approach consists of extracting the predictable movements from the observed data [22]. Computer software MINITAB version 16 was used for the estimation of statistically significant model parameters. The parameters show that the constructed model is significant at 6% level with χ^2 =15.0 (with 8 df). Partial Autocorrelation plot of residuals also suggest that the model is adequate in Figure **2**. In Table **1** forecast values of CO_2 concentration from ARIMA model. Next, we develop a regression model of traffic volume and CO_2 in urban Karachi.

3.1.1. Regression Model of Traffic Volume and CO_2 in Urban Karachi

We assume that the local CO_2 pollution fluctuation (*CO_2-concentr*), is a function of total urban traffic volume fluctuation, *total_traffic*, so that our model can be cast in the form

$$CO_2\text{-}concentr = \beta_o + \beta\ total\_traffic \quad (2)$$

the βs being as usual the regression model coefficients. Final model is given below.

$$CO_2\text{-}concentr. = 0.615 + 0.0000022*total\_traffic \quad (3)$$

In Tables **2** and **3** are depicts Model estimation and statistical results. Model (2) is a regression model between urban total traffic volume and traffic-related CO_2 emissions in Karachi. we estimate the concentration of CO_2 in the years 2007 and 2008 and compare with the forecast values given by ARIMA(2, 1, 2) in the previous section. The regression model gives

Table 1: Forecast Values of CO_2 Concentration from ARIMA (2, 1, 2)

Year	Forecast[in (μgm^{-3})]	Lower 95% Limits	Upper 95% Limits
2007	4.33	3.97	4.69
2008	4.38	3.87	4.89

Table 2: Regression Model (2) Between Urban Total Traffic Volume and Traffic-Related CO_2 Emissions in Karachi Estimated Parameters

Predictor	Coef	SE Coef	*T*	*P*
Constant(β_o)	0.61532	0.07572	8.13	0.000
β	0.0000022	0.00000009	26.38	0.000
	R-Sq	96.5%	R-Sq(adj)	96.4%

Table 3: Analysis of Variance of Regression Model (2)

Source	DF	SS	MS	F-test	P-test
Regression	1	15.115	15.115	696.17	0.000
Residual Error	25	0.543	0.022	S	0.1473
Total	26	15.658			

concentration of CO_2 4.13 μgm^{-3}, and 4.61 μgm^{-3} for the years 2007 and 2008 respectively which are very closed to the forecast values given by ARIMA (2, 1, 2).

4. CONCLUSION

As Section 1 demonstrates, greenhouse emissions from traffic flow in densely urbanized centres of the world is no less important than the contributions from industry, deforestation, etc., however, data collection in third world countries is sparse. This requires a clear methodology and development of appropriate indicators to handle the problem. Thus Section 2 addressed the problem of modeling the urban traffic-related CO_2 gas concentration fluctuations, with Karachi Region as a case study. As an application of the constructed traffic volume model, Section 3 developed an empirical model of urban traffic volume fluctuations. It revealed that by the year 2020 the total traffic volume in Karachi would reach to 6 million which might cause chaos in traffic control at that time and will in turn reduce the liveability ranking of the city. This section also developed a time series model for capture the fluctuating pattern of CO_2 gas concentration in the city. Finally, a regression model was obtained for CO_2 gas concentration as a function of urban traffic volume. As indicated by our preliminary calculations, the forecast values of CO_2 from the two models were comparable.

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